On the dynamics of growth and debt

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a post-Keynesian analysis

CASPER VAN EWIJK

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Voor Emma

CONTENTS

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CHAPTER 1

INTRODUCTION

1.1 GROWTH AND DEBT

Recent economic developments have been characterized by large shifts in the distribution of debt and wealth: on a country scale within the private sector and between the government and the private sector, and on a world scale among developed countries and between developed and less-developed countries. There are many indications that these shifts have exerted a pervasive influence on long-term movements in economic growth. Even the seemingly steady growth of the sixties was associated with significant shifts in financial positions. Behind the façade of continuing high growth in most European countries there was a gradual deterioration of the financial position of firms. That this initially did not lead to a slowing down of growth was due to a strong urge to grow, apparently based on over optimistic expectations about future demand and profits, and on an underestimation of the risk attached to it. When in the 1970's profits fell and risk increased, the financial position of firms proved too much weakened to absorb these shocks smoothly. As a result investment collapsed more than might have been expected on cyclical grounds alone.

A further complicating factor was that the slow-down of economic growth caused government budget deficits to rise, even without any discretionary expansionary fiscal policy. As these deficits had to be financed by issuing debt, the burden of interest payments increased as well, thereby raising the deficits even further. The fear of ever increasing interest payments and debt has become a serious embarrassment for 'Keynesian' demand management policies.

These events caused, both in theory and in practice, a shift in attention from the effects of demand management to the long-term implications of fiscal and monetary policy. Following the provocative analysis of Blinder and Solow (1973) there has been a wave of modelling of the dynamics implied by the government budget constraint (cf. Tobin and Buiter 1976, Turnovsky 1977, Christ 1978, 1979, Tobin 1982, Rau 1985 to mention but a few). Nevertheless, despite the intrinsically long-term nature of the accumulation of debt practically all these analyses start from more or less modified IS/LM models. However, in our opinion the basically static IS/LM framework is not very suitable for this analysis. As is well-known, the accumulation of debt and wealth is a very slow process which takes decades rather then years. Therefore it seems natural to start the analysis from a model which gives special attention to medium and long-term economic dynamics.

The IS/LM model has been developed for the analysis of short-term equilibrium. It concentrates therefore on the relations and mechanisms which are important from a short-term point of view, and it hardly pays any attention to processes that determine the evolution of the system in the longer term. Modifying the IS/LM model by just adding the government budget constraint while maintaining the basic structure of the model is in our opinion not sufficient to provide an adequate framework for analysing the dynamics of asset and debt accumulation. In most of the studies mentioned above this leads to the odd construction that the dynamics of debt and asset accumulation are analysed with reference to a stationary economy; i.e. an economy in which autonomous expenditure, labour supply and often even capital stock are supposed to be fixed.[1]

1.2 A POST-KEYNESIAN APPROACH

In this book we shall analyse the accumulation of debt and wealth starting from the post-Keynesian theory of growth and income distribution. This theoretical framework allows us to investigate the dynamics of government debt simultaneously with the evolution of the distribution of income and wealth between different classes. Moreover, it provides a good starting point to analyse the dynamic interaction between growth, asset accumulation and the fiscal policy of the government. The basic characteristics of the post-Keynesian approach are discussed briefly below.

Differential saving

One of the corner-stones of post-Keynesian theory is the classical proposition that on average a higher fraction is saved out of profits than out of wages. As a result, aggregate savings depend on the distribution of income over wages and profits. Whereas this proposition is "considered to be true by most macroeconomists" (Malinvaud 1986, p.110), it is neglected in the IS/LM model which assumes a uniform rate of saving for all types of income. This is unwarranted, especially in a long-term analysis with an endogenous distribution of wealth, because it conceals the role of distribution effects in the determination of the income-expenditure equilibrium.

Dynamic explanation of income distribution

A second characteristic of the post-Keynesian approach is the dynamic explanation of income distribution. This is often summarized in the statement that the profit rate is determined by the growth rate rather than vice versa (Kaldor 1966, Asimakopoulos 1986). This theorem distinguishes post-Keynesian theory from both Marxian theory,

[1] Cf. Blinder and Solow (1973) in their first model, Turnovsky (1976), Christ (1978,1979), Calvo (1985), Rau (1985).

according to which profits are determined by the surplus left by the given wage rate, and neoclassical theory, which explains the profit rate from the marginal productivity of capital. Basically, the post-Keynesian approach originates from a synthesis of the classical and Marxian notion that profits are the main source of savings, and the Keynesian proposition that in aggregate savings adjust to investment. Taken together these notions imply that profits will always adjust to the level just sufficient to provide the savings necessary for the financing of planned investment. This is the essence of Kalecki's famous "capitalists get what they spend" formula and Keynes' parable of the widow's cruse.

In the long term the distribution of income is determined by the dynamics of saving, investment and labour market disequilibrium. It cannot therefore be reduced to a simple (marginal productivity) rule of distribution. The profit rate is in post-Keynesian theory essentially a dynamic concept.

Choice of technique

It is often suggested that the assumption of an exogenous technique of production is also essential to post-Keynesian theory. Many post-Keynesian authors do indeed assume fixed coefficients of production, but certainly not all of them. For example, throughout the whole of his work Kaldor emphasizes the endogeneity of production technique.

As is argued by Darity (1981) the assumption of a fixed technique is not binding at all. The important point is that, even if one accepts a well-behaved production function, it does not automatically follow that income distribution is determined by the marginal productivities of labour and capital. As mentioned by Eichner (1985, p. 161), the distribution of income in post-Keynesian theory is explained "by a set of macroeconomic conditions rather than by the microeconomic factors emphasized in neoclassical theory." Ultimately, the problem is not so much whether or not a well-behaved production function exists, but rather whether the neoclassical state to which the marginal productivity theory of distribution applies has any theoretical or practical relevance (cf. Robinson 1971). Outside the (stationary) neoclassical state the causation runs from income distribution to the choice of technique rather than the other way around. In the present book it is therefore assumed that, if the technique of production is endogenous at all, it reacts only slowly to changes in factor prices. Besides, the optimum technique of production can be shown to depend more on variables other than factor prices alone as will be see in chapter 4.

1.3 METHOD AND SCOPE

The post-Keynesian theory of growth and distribution may be conceived within the sequential analytical approach (cf. Malinvaud 1980, Kuipers 1981) with three distinct levels of modelling. Each of these levels can in principle be investigated independently of the other levels.

The first level, the *short period*, refers to the determination of income-expenditure equilibrium. On this level capacity is given, and it is assumed that prices and wages are rigid and that equilibrium is achieved by quantity adjustment. This analysis builds on the 'classical' contributions of Kalecki (1933,1938,1943) and Keynes (1936).

On the *medium period* level income-expenditure equilibrium is taken for granted. Nominal wages are still assumed to be fixed, but capacity is now endogenous. As to the price level two views seem to exist. Some economists, such as Kalecki (1933,1935) and Harrod (1939), consider prices to be slow in comparison to the 'accelerator-multiplier' dynamics of investment and demand; while others, such as Robinson (1962) and Kaldor (1956, 1957) consider prices, and thus income distribution, as sufficiently flexible to avoid these quantity dynamics. In their view the medium period is thus characterized by the dynamics of growth and income distribution. In both approaches the central question is the same, namely whether medium-period dynamics leads to a steady (warranted) growth where expectations of demand and profits are continuously fulfilled. We shall follow the first approach and assume that the medium period is characterized by sluggish prices and disequilibrium between demand and capacity.

Finally, on the *long period* level both income-expenditure equilibrium and warranted growth are pre-supposed. Now wages and prices are assumed to be flexible. Also the distribution of wealth is now endogenous. The analysis then concentrates on technical change, on labour market disequilibrium and wages and on the accumulation of debt and wealth. The central question is whether the system tends to a 'golden age' with a constant rate of unemployment and a constant distribution of income and wealth. This level encompasses the analysis of the 'Marxian' dynamics of labour market disequilibrium and income distribution (following Goodwin 1967) as well as the analysis of Pasinetti processes of accumulating wealth and debt.

In some respects these levels are different from the levels distinguished by Malinvaud and Kuipers. Malinvaud concentrates on the short period and medium period levels and largely neglects the long period. Kuipers has extended Malinvaud's analysis to the long period, but defines this long period somewhat differently than we have done above. In the first place as he does not pay attention to the accumulation of financial assets, this criterion is invalid for his analysis. Kuipers distinguishes the long period from the medium period by the endogeneity of price-expectations, which he considers to be fixed in the medium period. Although we agree with Kuipers (and Malinvaud) that the adjustment of production technique to changes in factor prices is essentially a long-term mechanism, it is in our view not very probable that this is caused by rigidity of expectations on factor prices. Instead, it seems more natural to explain the rigidity of production technique by adjustment and development costs. Following

Malinvaud (1980, p.11) we consider lagged adjustment of (price) expectations as a medium-term phenomenon.

This book concentrates on the medium and long period level. The short period is neglected in most of our analysis. Not because it is unimportant or uninteresting, but because of the simple fact that it takes us too far from our main concern: the long-term evolution of debt and wealth positions.

1.4 WHY STUDY LONG-TERM DYNAMICS?

In a world characterized by a succession of shocks and fluctuations one might wonder, remembering Keynes, why should one be concerned about the long run? As will be seen the process of changing debt and wealth normally takes place slowly; after a disturbance of long-term equilibrium it may take many decades before a new steady situation is reached. Nevertheless, these long-term processes may play an important role in the modelling of the medium and the short period. Besides the argument that "growth models ... should be introduced in any satisfactory theory of medium-term evolution" for reasons of analytical clarity and robustness (Malinvaud 1980, p.11), it also seems natural to assume that the long-term consequences of current decisions are taken into account when making these decisions. For instance, if one is aware that the cumulative process of growing debt and interest payments will sooner or later accelerate and grow out of control, it may be sensible to take on no debt at all, even if one knows that the growth of debt may in principle be sustained for a long time.

However, as has become clear from recent theories on individual behaviour, it is impossible to calculate the risk-discounted optimum when the economy is characterized by true, Keynesian or Knightian uncertainty[2] (cf. Weintraub 1979, Blatt 1983, Eichner 1985). In that case one has to revert to more conventional decision rules. The emphasis on uncertainty as opposed to risk (that is 'uncertainty' with a known probability distribution) is a distinctive feature of post-Keynesian (microeconomic) theory in comparison with the neo- and new-classical economics.

In Keynes view uncertainty is especially crucial with respect to decisions concerning the accumulation of wealth: "The whole object of the accumulation of wealth is to produce results, or potential results, at a comparatively distant, and sometimes at an *indefinitely* distant, date. Thus the fact that our knowledge of the future is fluctuating, vague and uncertain, renders wealth a peculiarly unsuitable subject for the methods of the classical economic theory" (Keynes 1973, p. 113).

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[2] In post-Keynesian literature, Keynesian and Knightian uncertainty are often lumped together under the title 'Keynes-Knightian' uncertainty. According to Hoogduin this is not justified: "Whereas Knight mainly focuses on the distinction between numerically measurable and not numerically measurable probabilities, Keynes stresses the slight amount of knowledge on which probabilities often have to be based" (Hoogduin 1987, p. 53).

This may explain why agents when deciding on debt and savings often adopt certain rules or norms on 'proper' financial behaviour. Despite the Modigliani-Miller hypothesis on the irrelevance of the method of financing investment, it is well-established that firms hold on to certain norms for their external debt as a guideline for their investment and financial decisions. Similarly, despite the Barro hypothesis on the neutrality of government finance (the 'Ricardian' doctrine, Tobin 1980) there exists an ongoing discussion on the proper norms for the government budget deficit. Finally, norms exist also with regard to the external position of countries as a whole. In the Netherlands we have long been accustomed to norms for the structural government budget deficit (3 to 4 per cent of national income) and the desired excess in the balance on current account (not less than 1 percent of national income).

Such behaviour, which is familiar in practice, seems at variance with the standard neoclassical conception of the rationally thinking and calculating homo economicus. The emphasis on convention and norms rather than on sophisticated risk- and time-discounted optimality calculations in the explanation of economic behaviour in general, and of investment behaviour in particular, has always been one of the principal themes of post-Keynesian theory. A well-known example is Kaldor's pay-back period in his explanation of investment. As has recently been shown by Blatt (1983), a modified pay-back rule may indeed offer a better criterion for investment than the standard (neoclassical) present value criterion when the economy is characterized by Keynesian or Knightian uncertainty. Similar views are found with regard to pricing and the financing of investment in Wood (1975), Eichner (1976), Cornwall (1983), Moss (1984).

An additional reason for the use of rules and norms is related to the decision-making process. Especially in organizations with a complex and costly decision making process, rules and norms may offer an attractive 'second-best' solution to discretionary decision-making. This even more so, if due to Keynesian or Knightian uncertainty no precise (numerical) assessment can be made of the costs and benefits of certain decisions. In these circumstances, a rule which is at least consistent from a long-term point of view, is often preferred to a continuous process of discretionary decision-making.

The stress laid on norms does not mean that behaviour is fixed for ever. It is evident that in the long term these norms must be founded on an economic assessment of costs and benefits. Thus, as has been shown by Blatt, the length of the pay-back period is also ultimately related to average risk and profitability. Therefore, in order to explain why short-term behaviour follows certain decision rules, it is necessary to view this against the background of a long-term theory explaining the rules. As a corollary, a major change or break in economic development will lead to a revision of all current norms and rules. Consequently, the short-period model will change as well.

In the three levels of analysis distinguished, there is thus a certain hierarchy.

Short-period behaviour is not truly independent of the long-period consequences of this behaviour. Short-period dynamics is therefore contingent on the set of rules and norms based on long-period considerations.

1.5 OUTLINE OF THE BOOK

Chapter 1 begins with a brief review of alternative theories of differential saving, and then as a first step in the analysis investigates the consequences of the introduction of the government budget constraint in a generalized Pasinetti–Kaldor model. It will be shown that this model in its 'classical' post-Keynesian form yields a highly unstable dynamic system, which can produce a stable 'two-class' solution only under very special, unrealistic assumptions.

In order to develop a more sophisticated macroeconomic model chapters 3 and 4 consider the long-term determinants of growth on the basis of a microeconomic model of a representative (corporate) firm. Following our main interest, this model concentrates on the long-term relationships between profits, finance and the growth of the firm. Chapter 3 develops a basic model for an equity-rationed 'managerial' firm. It is shown that this firm faces a trade-off between growth and risk. This 'growth–risk' frontier is the basis for the determination of the optimum rate of growth. Chapter 4 relaxes the assumption of full equity-rationing and introduces an (imperfect) equity market. As a result the conflict of interests between managers and shareholder must be taken into account too. Furthermore, this chapter introduces costs of growth and examines the consequences for the growth strategy and the adjustment process.

Thereafter we shall return to the macroeconomic level and complete our model for the closed economy. This model will be used to investigate the dynamic relationships between growth, income distribution and the financial positions of the government, firms and the social classes distinguished. Particular attention is given to the stability of the system for alternative fiscal and monetary regimes. Chapter 5 concentrates on the medium period and chapter 6 on the long period.

In chapter 7 the analysis will be generalized for the open economy. Because in an open economy the equilibrium generating role of income distribution is impeded by the existence of foreign competition in goods and financial assets, we shall concentrate on the balance of payments as an outlet for internal disequilibrium between investment and saving. As a consequence the dynamics of the government budget constraint is intimately linked to the dynamics implied by the balance of payments constraint; that is, one should consider the development of public debt in relation to the evolution of the net foreign creditor or debtor position. This chapter will be concluded by some considerations with respect to economic policy in the presence of external and internal disturbances.

Our method is mainly theoretical. Where necessary mathematical techniques will be used to assess the dynamic characteristics of the models to be developed. Each model will be considered in turn to establish whether a unique path of steady growth exists or not. Next the dynamics of the models will be examined, as far as possible by analytical means and in most cases illustrated by numerical simulations. Furthermore, when possible, the impact of the choice of the parameters of the model on the general stability of the system will be assessed as well as the impact of alternative fiscal and monetary regimes.

CHAPTER 2

PASINETTI PARADOXES AND THE GOVERNMENT BUDGET CONSTRAINT

2.1 INTRODUCTION

This chapter examines the long-term dynamics of public debt in a two-class post-Keynesian model. Our concern is twofold. In the first place we are interested in the dynamics arising from the government budget constraint when analysed on the basis of a post-Keynesian model where the distribution of income between wages and profits is the key mechanism for savings-investment equilibrium. This point of departure of our analysis is radically different from most other macroeconomic investigations of the dynamics of public debt which adopt the IS/LM framework in which income distribution plays no (explicit) role at all.[1]

Secondly, we wish to examine the consequences for the consistency of the post-Keynesian model when the government budget constraint is introduced. As is well-known, Pasinetti's model, as well as more general models based on it, is subject to rather stringent conditions which must be satisfied for an interior 'two-class' solution to be feasible.[2] If these conditions are not satisfied, one of the social classes will disappear in the long run, and what is more serious, the distribution of income over workers and 'rentiers' will no longer operate as a mechanism ensuring equilibrium between savings and investment. The possibility of such an 'anti-Pasinetti' state led Samuelson and Modigliani (1966) to conclude that a more general theory of income distribution is required, based on the principles of the neoclassical marginal productivity theory. We shall re-examine this conclusion with reference to the generalized post-Keynesian model to be developed in this chapter.

The chapter is built up as follows. First, there is a brief review of several alternative views on differential savings (section 2.2). Particular attention is given to the role of retained earnings (section 2.3). This discussion of differential savings provides the starting point for our modified post-Keynesian model (sections 2.4-2.7). The basic difference with traditional post-Keynesian models concerns the inclusion

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[1] Among the classical papers on this subject are Christ (1968), Blinder and Solow (1973) and Tobin and Buiter (1976).

[2] The debate was initiated by the famous article of Pasinetti (1962) and its discussion by Samuelson and Modigliani (1966). There have been many attempts to generalize Pasinetti's model and to criticize the conclusions of Samuelson and Modigliani with reference to their 'anti-Pasinetti land,' see, for instance, Laing (1969), Chiang (1973), Pasinetti (1975, 1983), Baranzini (1975), Fazi and Salvadori (1981), Darity (1981), O'Connell (1985).

of the government sector. In most other respects it follows the 'classical' post-Keynesian model, assuming constant propensities to save, a fixed technique of production, a given ('natural') rate of growth, etc. Some of these assumptions will be relaxed in section 2.8 when wealth and interest effects are introduced in the saving function and the technique of production is related to factor prices. However, the analysis stays confined to the evolution of debt and wealth in a steady state world with a given 'natural' rate of growth. The dynamics of investment and income distribution, the role of money and the international aspects are considered in later chapters.

2.2 A BIRD'S EYE VIEW OF DIFFERENTIAL SAVING

One of the basic theorems of post-Keynesian theory is the proposition that savings-investment equilibrium is ensured by shifts in the distribution of income between wages and profits. According to Robinson (1956) excess demand for goods causes sellers' markets to emerge with rising prices and profit margins, and correspondingly falling real wages. As the (marginal) propensity to consume out of profits is supposed to be less than the propensity to consume out of wages, this shift in income distribution reduces aggregate demand and thus restores equilibrium between aggregate demand and supply. In the opposite case of excess supply, buyers' markets will arise, leading to falling profits and rising real wages, and thus to a rising demand.[3]

Other authors have suggested that there also exists a direct link between profit margins and (planned) investment (cf. Eichner 1976, Harcourt and Kenyon 1976). Building on Kalecki's mark-up hypothesis they argue that rapidly growing firms will set higher profit margins than similar firms with a low rate of expansion because they have a larger need for investment funds.[4] Thus in this theory also, investment

[3] Actually this mechanism requires considerable homogeneity of consumption and investment goods, as otherwise it is difficult to understand how an increase in demand for investment goods leads to a rise in prices of consumption goods, and thus to lower real wages. In 'Accumulation of Capital' (1956) Robinson is much more prudent on this point than in many later (one-sector) representations of the post-Keynesian model.

[4] Eichner motivates this relation between pricing and investment on the basis of a smaller price-elasticity of demand for the firm's products in the short term than in the long term. This implies that the firm can increase its net returns in the short term by raising its profit margin at the expense of lower returns in later years. Therefore it is attractive for firms to raise their profit margin as long as the implicit costs of this internal method of finance are less than the costs of external finance. In the case of an upward sloping supply curve of external finance it is obvious that the optimum profit rate is positively associated with the volume of investment.

Note that the underlying trade-off between short-term and long-term price-effects on demand is very similar to the well-known J-curve effect of the balance of payments. Eichner's theory is thus built on a 'J-curve effect' in net returns and a (non-Modigliani-Miller) rising supply curve of external finance.

generates (part of) its own savings.[5]

It is obvious that this post-Keynesian distribution mechanism hinges on two propositions: the sluggishness of nominal wages and the higher propensity to save from profits than from wages. In most of the (older) post-Keynesian theories, nominal wages are therefore taken to be given historically. Robinson supposes that workers do not resist inflationary erosion of their income as long as the share of wages is not pressed beyond some minimum (the 'inflation barrier').[6] With regard to the proposition of differential savings, the post-Keynesian theory evidently builds on the classical theories of Ricardo and Marx, according to which workers consume the whole of their income, while capitalists (and 'rentiers') save the larger part of their income. In Marx' view wages serve for the reproduction of labour power only, and thus leave no room for saving, while profits in the capitalist system provide both the motive and the finance for the accumulation of capital.

Life cycle-hypothesis

It needs little argument to show that this class-related explanation of differential saving is at variance with mainstream (micro)economic theory which takes the individual as the unit of analysis, and denies the existence, or relevance, of social classes. It is often thought, and claimed, that the life-cycle theory of saving has finally rejected the 'outdated' idea of differential saving. As, for any individual, saving is just deferred consumption, there is, according to this theory, no reason to assume that consumption is systematically higher for one group than for another group. Whether income is high or low, individuals aim at an optimal spread of consumption over their lifetime. In fact, a shift in distribution from wages to profits may even produce a (temporary) *reverse* effect on savings: a rise in profits increases the income of the pensioned people, who consume their full income or even dissave, while the fall in wages affects the younger people who are still saving for their own old age. This shift in the intergenerational distribution of income thus gives rise to a '*perverse neoclassical saving function*' (Marglin 1984, p.45).

However, this representation of the life-cycle model is too simple; it neglects the existence of liquidity constraints and of intergenerational transfers, gifts and bequests. If these aspects are taken into account as well, the life-cycle model can be shown to be fully consistent with differential saving, and even to give a justification for it. For example, it can be shown that it is sufficient to introduce two classes (or groups)

[5] There is no reason to assume that this mechanism is in itself sufficient to generate all the savings required, unless one follows Pasinetti (1981) and presupposes full internal financing. This theory is therefore less general than Robinson's mechanism sketched above. Moreover, this mark-up mechanism applies exclusively to investment; if excess demand arises from sources other than investment, such as government expenditure, private consumption or exports, this mechanism fails completely.

[6] This term was first introduced in Robinson (1956). A similar process was described earlier by Kalecki (1954, p.48).

with different attitudes towards inheritance. (See *Appendix 2.A* for a proof on the basis of a simple two-class life-cycle model).

Macroeconomists' view

Contrary to the above negative (microeconomic) view on differential saving is the much more positive standpoint of most macroeconomists. According to Malinvaud (1986, p.110) "the prevailing [macroeconomic] view is definitely in favor of the truth and significance of the proposition." Therefore it is, in his view, necessary to reconsider the significance of differential saving and to work on a justification from the mainstream point of view. Malinvaud distinguishes three possible (complementary) explanations for the proposition that the marginal propensity to save is less for wages than for profits:

1. *Pure distribution effect*: Wage-earners generally face stronger liquidity constraints than profit-earners, so that they react more strongly to changes in their actual income than profit-earners. Moreover, because profits have a higher variability than wages, a wage change is considered to be more permanent and thus has a stronger effect on consumption than the opposite change in profits.
2. *Retained earnings*: Because of the 'corporate veil' and perturbations in the stock market valuation of shares, a change in corporate retained earnings is generally valued less than a similar change in wages or distributed profits.
3. *Differential taxing*: When profits are more heavily taxed than wages a shift from wages to profits leads to higher aggregate savings.[7]

At this stage it is important to note that these explanations of differential saving all relate to differences in the type or source of income. No attention is paid to the fact that differences in savings propensities may also arise from different attitudes amongst the recipients of the income, i.e. amongst different social classes. In this respect they are in contrast to the classical explanation discussed above.

Kaldor and Pasinetti revisited

A similar conflict in the explanation of differential saving is found within the post-Keynesian school between Kaldor and Pasinetti. While Pasinetti adheres to the classical proposition that differences in savings propensities should be related to social groups with different attitudes towards savings, Kaldor explicitly rejects this idea and regards "the high savings propensity out of profits as something which attaches to the nature of business income" (1966, p.310). Thus, in contrast to Pasinetti who attributes the higher propensity of rentiers to save to "a stronger tendency for people in the higher income brackets (normally wealth owners) to plan for inheritance" (1983, p.100), Kaldor explains the higher savings out of profits from the necessity of

[7] This is true only if subjects do not see through the 'government veil.' According to the well-known 'Ricardian equivalence' theorem, changes in government savings are fully compensated by private savings if all subjects take full account of future taxes (cf. Barro 1974, Tobin 1980).

corporate firms " to plough back a proportion of the profits earned [...] in order to ensure the survival of the enterprise in the long run" (1966, p.310). In a formal manner the alternative saving functions may be modelled as:

$$S = s_1(y-\pi) + s_2\pi \qquad \text{(Kaldor)} \qquad (2.1a)$$

$$S = s_w(y-\pi+z_1\pi) + s_c(1-z_1)\pi \qquad \text{(Pasinetti)} \qquad (2.1b)$$

where $0 \leq s_1 < s_2 \leq 1$ and $0 \leq s_w < s_c \leq 1$, and
z_1 = share of workers in total wealth
π = rate of profit
s_1, s_2 = savings propensities from wages and profits
s_w, s_c = savings propensities of workers and capitalists
S = aggregate saving (ratio to capital stock)
y = production (ratio to capital stock)

Throughout the following all stock and flow variables are expressed as ratios to capital stock. The first equation relates savings propensities to wages $(y-\pi)$ and profits π, whereas the second equation relates them to the income of workers, including their share in profits $(y-\pi+z_1\pi)$, and the income of rentiers $(1-z_1)\pi$. For simplicity it is assumed that s_w=s_1 and s_c=s_2 and s_1 and s_2 are used for s_w and s_c below.

In the short run, when the distribution of wealth is given, these functions are not essentially different from a macroeconomic point of view. In both cases aggregate savings are positively associated with the share of profits; only the slope is different.

$$\frac{dS}{d\pi} = (s_1-s_2) \qquad \text{(Kaldor)} \qquad (2.2)$$

$$\frac{dS}{d\pi} = (1-z_1)(s_1-s_2) \qquad \text{(Pasinetti)}$$

However, despite this short-term similarity there is an important difference between these two saving equations from a long-term point of view. Because savings add to wealth one must in the long-period take account of the evolution of the distribution of wealth as well. This has led Pasinetti to his famous signalling of a logical 'slip' in Kaldor's model, because it allows for the savings of workers but disregards the accumulation of wealth ensuing from it. Therefore, Pasinetti considered his function as a 'repaired' version of Kaldor's.

Kaldor did not accept Pasinetti's model as an improved version of his own model. Nor did he agree with the 'schizophrenic' interpretation of his savings function suggested by Meade (1963), Samuelson and Modigliani (1966). According to this interpretation workers have different savings propensities with regard to wages and to profits.[8] As mentioned above, Kaldor argues that the higher savings propensity out of profits must be explained by the fact that not all profits are distributed to its formal owners, the shareholders. Unfortunately, Kaldor never worked out this view more explicitly.[9]

A generalized savings function

There have been many attempts to integrate Kaldor's idea of undistributed profits with Pasinetti's long-term model in order to obtain a 'general' post-Keynesian model (e.g. Chiang 1973, Darity 1981, Marglin 1984). These attempts generally lead to a mixed savings equation like

$$S = s_1(y - \pi + \theta\pi z_1) + s_2\theta\pi z_2 k + (1-z\theta)\pi \tag{2.1c}$$

where θ = fraction of profits distributed ($0 \leq \theta \leq 1$)
z_1 = share of workers in total capital
z_2 = share of rentiers in total capital
$z = z_1 + z_2$

Usually it is assumed that $z=1$, which means that workers and rentiers own the whole capital stock. Although this is not strictly necessary, as will be seen below, we shall for the moment follow this common proposition. This generalized function (2.1c) is 'Kaldorian,' as the effective savings rate of workers is different for wages and profits, s_1 and θs_1 respectively, while at the same time it is 'Pasinettian,' as it distinguishes two classes with different propensities to save, s_1 and s_2. If all profits are distributed ($\theta=1$) and $z=1$ this equation reduces to the simple Pasinetti equation (2.1b). If, alternatively, savings propensities are identical for both classes ($s_1=s_2$) we obtain a quasi-Kaldor function where the propensities to save out of wages (s_1') and profits (s_2') are equal to

$$s_1' = s_1$$

$$s_2' = s_1 + (1-s_1)(1-\theta)$$

Whenever profits are not fully distributed ($\theta<1$) this result implies that $s_2'>s_1'$.

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[8] A modern motivation for this interpretation may be found in the argument of Malinvaud (1986) that the higher savings propensity of profits may be due to their higher variability (see above). In our view this explanation is valid in the short term only; in the long run subjects are able to discern between temporary and permanent shifts in profits and adjust their consumption accordingly.

[9] It is often suggested that Kaldor's neo-Pasinetti theorem is an elaboration of this idea. However, the mechanism underlying this theorem is totally different from the post-Keynesian income-distribution mechanism. Instead of wages and profits it attributes a central role to the valuation of shares in maintaining income-expenditure equilibrium. In essence it is a sophisticated version of the common interest mechanism with respect to savings-investment equilibrium showing a close resemblance to Tobin's q hypothesis.

There exist, however, some fundamental problems with regard to this generalized function that often seem to go unnoticed. These problems arise from the ambiguous interpretation of the fraction of profits distributed θ. Basically, there are two alternative interpretations: one relating to different returns on financial assets and capital stock, and the other relating to retained earnings.

1. According to the first interpretation, θ measures the ratio between the rate of return on the financial wealth of workers and rentiers and the real profit rate. As has been suggested originally by Laing (1969) and elaborated by Pasinetti (1975, 1983) and Fazi and Salvador (1981) the reward on financial assets of workers (and rentiers) is generally lower than the return on capital. However, if one adopts this interpretation of θ, the question arises where the difference between total profits and distributed profits remains. In fact, this interpretation is only consistent if one allows for a third class, namely the capitalists, who organize the production and to whom accrue the undistributed profits. In that case, however, it can no longer be automatically assumed that the shares of workers and rentiers in total wealth sum up to unity. On the contrary, one should also allow for the accumulation of wealth by the capitalists (hence $z<1$). If this is not recognized properly one falls into a 'slip' similar to the one as Pasinetti blamed Kaldor for.

2. The second interpretation of equation 2.1c is that workers and rentiers do indeed own the entire capital stock, and that profits have a smaller impact on consumption because only a fraction of profits is distributed to the shareholders. This interpretation seems more in accordance with Kaldor's views and fits in with Malinvaud's second ('corporate veil') explanation above. However, it is not right in that case to conceive of θ as the fraction of distributed profits as Chiang (1973), Darity (1981) and Marglin (1984) do, for this would imply that shareholders do not receive any capital gains at all, or alternatively, that capital gains have no effect on their consumption. Retained earnings are thus simply assumed to vanish. This is clearly unwarranted with reference to modern society where most of the savings of workers and rentiers consists of claims to pension funds and life assurance. The next section takes a somewhat closer look at the impact of retained earnings on consumption and savings.

2.3 A DIGRESSION ON RETAINED EARNINGS AND AGGREGATE SAVINGS

As is known, the income of shareholders consists of dividends and capital gains. If we express the total value of shares as a ratio q to capital stock net of borrowing (1-a) total income on shares y_s may be represented as

$$y_s = \delta + jq(1-a) \tag{2.3}$$

where δ stands for the pay-out of profits (dividends), j for the rate of increase of total share value and a for (net) external debt of the corporate sector. Adding this share-income to wage-income and interest-income we obtain for total income perceived by workers and shareholders y_p

$$y_p = y - \pi + ra + \delta + jq(1-a) \tag{2.4}$$

In order to concentrate on the role of retained earnings we assume for simplicity that the firm keeps its debt ratio constant and does not issue new shares. Then the budget constraint of the corporate sector is[10]

$$\pi - \delta - ra - (1-a)i = 0 \tag{2.5}$$

where i stands for net investment. Since i is expressed as a ratio to capital stock it stands for the growth rate of capital stock as well. After substitution of this constraint into equation (2.4) perceived income becomes

$$y_p = y + (jq-i)(1-a) \tag{2.6}$$

Note that y_p is independent of the pay-out rate (δ); this implies that no income vanishes directly as a consequence of corporate retention. Nevertheless perceived income may be different from actual income (=y) if the growth of share value differs from the growth of capital stock ($jq \neq i$). This may occur if $q \neq 1$ or $j \neq i$; that is, if capital stock or its rate of growth are under- or overvalued.

Now consider the relation between income distribution and aggregate savings. Following Kaldor it is assumed that there is one, non-schizophrenic, class which saves a given fraction (s) of perceived income irrespective of its source. Nevertheless, a shift in income distribution may have an influence on aggregate savings, namely if it gives rise to a change in perceived income. Thus, since consumption equals $(1-s)y_p$ aggregate savings (S) is given by

$$S = y - \text{consumption} = sy - (1-s)(jq-i)(1-a) \tag{2.1d}$$

This equation shows that aggregate savings differ from planned savings (sy) by a factor measuring 'unintended' consumption due to the wrong perception of corporate income. If jq exceeds i capital gains are overvalued, so that perceived income exceeds actual income. As a result consumption will be greater, and savings correspondingly lower.

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[10] This equation follows from the budget constraint $Da=\pi-\delta-ra-(1-a)i$ and the condition of a constant debt ratio $Da=0$, where $Da=da/dt$. This assumption eases our exposition; it is not essential to our argument.

In order to examine whether savings are positively related to the share of profits, one should look in more detail at the determination of the factor (jq-i). Imagine that at a certain moment real wages fall and profits rise while all other factors remain unchanged. If firms hold on to a given debt ratio they must either raise the pay out of profits or speed up their rate of investment. In the first case the actual income of shareholders increases by the same amount as wages have fallen, so that total income is unchanged. However, if the rise in profits is thought to be permanent, the valuation of shares will be revised upward, thereby causing an increase in perceived income. As a result the higher profits now lead to a *fall* in aggregate savings. This effect is contrary to the anticipated effect we were seeking: a shift in income distribution causes a *reverse Cambridge-effect.*

If, alternatively, the extra profits are used to increase investment the sum of wages and distributed profits will indeed fall. Then whether total perceived income will fall or rise depends on the degree to which the growth in capital stock is reflected in the growth of share value. In the short run this depends on many expectational and psychological factors, which we shall not try to unravel here.[11]

It is evident that the mere existence of retained earnings does not provide a sufficient justification for the post-Keynesian proposition of differential saving. Some other factors have to be taken into account as well. One might follow Malinvaud and add liquidity constraints and informational imperfections to the model. Because this approach stresses disequilibrium and market imperfections as the principal cause of differential or 'forced' savings, it seems more appropriate in the short period than in the long period.

As our main interest is the long-term evolution of savings and wealth, in the subsequent section another approach will be used to explain, following Pasinetti, differential saving in terms of different attitudes towards saving between social classes. However, we do not agree with Pasinetti's view that the propensity to save is a purely psychological concept which has to do with the attitude towards provisions for old age and inheritance only (Pasinetti 1983, p. 100). In this respect we have sympathy for the arguments of Kaldor (and Marx) that differences in savings propensities also arise from the institutional organization of the economy. The basic error of Kaldor's model is, however, that it rejects the idea of social classes altogether and conceives the corporate sector as an anonymous and non-owned Moloch who absorbs retained earnings without ever giving anything back in return. As we have seen this view is unwarranted. Even in modern society there exists a class of managers and owners with a distinct role in the economic process and with a different attitude towards saving from the workers. For workers savings concern primarily deferred consumption and provision for old age, and for the better-off employees maybe for

[11] See Malinvaud (1986) for a discussion of these short-period factors.

inheritance as well. For the class of owners and top-managers saving is, however, also a means for maintaining and, if possible, increasing their power and status. They save for the intrinsic benefits of wealth as well.

Therefore the subsequent analysis adopts a savings function with two different classes, workers who receive interest on their savings and a corporate class of owners and managers who appropriate profits after payment of interest to the workers. As the interest rate is in general lower than the profit rate, this conception allows for differential rewards on savings by workers and on the wealth of the owners of the capital stock. As all profits, after payment of interest, accrue to the corporate class there are no vanishing retained earnings.

2.4 THE PASINETTI PARADOX

As was shown above (section 2.2) the Pasinetti and Kaldor savings equations are not essentially different from a short-term point of view. However, in long-term equilibrium when the distribution of wealth too is endogenous, these functions yield remarkably different results[12]

$$\pi = \frac{i-s_1 y}{s_2-s_1} \qquad \text{(Kaldor)} \qquad (2.7a)$$

$$\pi = \frac{i}{s_2} \quad \text{and} \quad z_1 = \frac{s_1}{i}\,\frac{s_2 y-i}{s_2-s_1} \qquad \text{(Pasinetti)} \qquad (2.7b)$$

As we have mentioned as investment (i) is expressed as a ratio to capital stock it represents the growth of capital stock as well. The result (2.7b) for Pasinetti's model has raised much controversy as it suggests that the long-term profit rate is independent of saving by workers. This is known as the *Pasinetti paradox.*[13]

The solutions for both the Kaldor and the Pasinetti model are, however, subject to rather stringent boundary conditions arising from limits for the distribution of

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[12] These equations are obtained by solving the conditions for flow equilibrium i=S and stock equilibrium $Dz_1=s_1(y-\pi+z_1\pi)-iz_1 = 0$.

[13] For a proper understanding of this result it must be recognized that this equation is merely a condition and not a causal relationship. In fact, the steady state profit rate depends on both the propensity to save of workers and of rentiers, as well as on the distribution of wealth, thus rewriting (2.7b):

$$\pi = (i-s_1 y)/\{(s_2-s_1)(1-z_1)\}$$

In the steady state the distribution of wealth turns out to be precisely such that this relation after substitution for the steady state share of wealth (z_1) produces equation (2.7b).

income.[14] It is evident that profits cannot fall below zero. For wages also there will exist a positive minimum (inflation barrier). In addition Pasinetti's model requires that the wealth of rentiers should not be negative. In summary these boundary conditions for both models can be written as

$$0 \leq \pi \leq (1-\xi)y \qquad (2.8)$$
$$z_1 \leq 1$$

where ξ stands for the minimum share of wages. Solving these conditions we find

$$s_1 y \leq i \leq \{\xi s_1 + (1-\xi)s_2\}y \qquad \text{(Kaldor)} \qquad (2.8')$$
$$s_1 y \leq i \leq (1-\xi)s_2 y \qquad \text{(Pasinetti)}$$

This result brings out that the boundary conditions are more restrictive for the Pasinetti model than for Kaldor's whenever $\xi s_1 \neq 0$.

These conclusions are corroborated if, not wages, but the *total* income of workers is bound by a minimum. In the present model, where a considerable part of workers' income may consist of interest income, this alternative condition might, from a theoretical point of view, be more appropriate than Robinson's inflation barrier which applies to wages only. Denoting the minimum income share of workers by χ, the boundary conditions become

$$s_1 y \leq i \qquad \text{(Kaldor)} \qquad (2.8'')$$
$$s_1 y \leq i \leq \{\chi s_1 + (1-\chi)s_2\}y \qquad \text{(Pasinetti)}$$

The lower boundary for i is again the same for both models, but the upper boundaries have changed significantly. Kaldor's model no longer yields an upper boundary for i at all. This is not surprising if one remembers that in Kaldor's model all income accrues to the workers either as wages or as profits. The share of income of workers is thus always unity. Likewise in the Pasinetti model the upper boundary proves to be less restrictive.

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[14] The stringency of the boundary conditions has been raised originally by Tobin (1960) with respect to Kaldor's model and by Samuelson and Modigliani (1966) for Pasinetti's model.

2.5 A GENERALIZED MODEL WITH GOVERNMENT

The introduction of the government sector has several important consequences for the post-Keynesian model. In the first place it adds a new possible cause of differential saving, namely different tax rates for wages and profits. If profits are taxed at a higher rate than wages, a redistribution from wages to profits will increase aggregate savings even if there is no difference in the propensities to save. This was also noted by Malinvaud (1986) (see section 2.2). Secondly, it complicates the long-term dynamics of the model as the accumulation of public debt must be taken into account as well.

The model is set up as follows. Besides the government sector there exist two classes: workers who receive wages and interest on their accumulated savings, and capitalists who appropriate the difference between profits and interest paid out to workers. One part of this residual consists of the return on their financial assets and the other part consists of an 'entrepreneurial' reward for organizing the production and bearing the risks attached to it. In order to avoid the complications connected with the valuation of shares workers are supposed not to own shares, but loans only.

The government sector receives taxes and finances its deficit by issuing loans to the private sector. Taxes are supposed to be levied at fixed rates on wages, interest income of workers and net earnings of capitalists. Expenditure of the government is taken as a constant fraction of total production. There is no government production.

In all other respects our model follows the 'classical' post-Keynesian model: the rate of growth is determined by the 'natural' rate of growth, the technique of production is fixed and distribution of income between wages and profits always ensures savings-investment equilibrium. Then expressing all stock and flow variables as ratios to capital stock the model can be written as follows

$$S = s_1\{(1-\tau_0)(y-\pi) + (1-\tau_1)(a+b)r\} + s_2(1-\tau_2)(\pi-ar) \tag{2.1e}$$

$$T = \tau_0(y-\pi) + \tau_1(a+b)r + \tau_2(\pi-ar) \tag{2.10}$$

$$i + g + rb = S + T \tag{2.11}$$

$$\pi = r + \phi i \tag{2.12}$$

$$D(a+b) = s_1\{(1-\tau_0)(y-\pi) + (1-\tau_1)(a+b)r\} - (a+b)i \tag{2.13}$$

$$Da = -s_2(1-\tau_2)(\pi-ar) + (1-a)i \tag{2.14}$$

$$Db = g + rb - T - ib \tag{2.15}$$

where s_1 and s_2 are the savings propensities of workers and capitalists respectively, τ_0, τ_1, τ_2 the tax rates on wages, interest income of workers and net earnings of capitalists, T total taxes, g government expenditure, b government debt, a the net debt of capitalists and r the (real) interest rate. The subscripts 1 and 2 refer to workers and capitalists. The D operator represents the first derivative with respect to time. Throughout the following it is assumed that workers save less than capitalists: $s_1 < s_2$.

The savings equation 2.1e differs from the simple Kaldor and Pasinetti equations (2.1a and b) in two respects. First, it distinguishes between the profit rate on capital (π) and the rate of return on financial wealth (r), and secondly, it takes account of (differential) tax rates on wages, interest income and net corporate earnings. Equation 2.10 defines total taxes. Equation 2.11 represents the condition for equilibrium between investment and savings. Equation 2.12 states that the profit rate is equal to the interest rate plus an entrepreneurial premium which is related to the (given) rate of growth (=i). This equality can also be interpreted as the condition for growth equilibrium, defining the profit rate in relation to the interest rate which is necessary to sustain a certain growth rate i.[15] It is not unreasonable to assume that the profit rate should be higher if the growth rate is higher. For the moment it is not necessary to explain how this condition is ensured, we simply assume that it is.[16]

These relations together form the static part of the model. The budget constraints for workers (2.13), capitalists (2.14) and the government (2.15) complete the model; they determine the dynamics of the system over time. Of course, only two of these differential equations are independent.

Statics

The static part of the model (equations 2.1f to 2.12) yields the following solution for the interest rate

$$r = \frac{x}{(c_0-c_1) + (c_1-c_2)(1-a) - c_1 b} \qquad (2.16)$$

where $x = i + g - (1-c_0)\, y - (c_0-c_2)\phi i$

$c_i = (1-\tau_i)(1-s_1)$ for $i = 0,1$

$c_2 = (1-\tau_2)(1-s_2)$

For the sake of brevity the symbol c_i is introduced representing the rate of consumption per category of income. Note that the denominator of (2.16) represents the effect of the interest rate on aggregate savings (dS/dr). Throughout the following

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15 This equation follows from the investment function $i = (\pi - r)/\phi$ and the condition for growth equilibrium according to which i must be equal to the given ('natural') rate of growth.

16 The determination of growth will be dealt with extensively in chapters 3 and 4. In chapters 5 and 6 we shall discuss how the 'natural growth' equilibrium is achieved. An intuitive explanation for eq. 2.12 is that a higher interest rate causes investment and growth to slow down, thereby leading to higher unemployment, and thus to lower wages until the profit rate has increased sufficiently to maintain the original rate of growth.

the analysis it is assumed that dS/dr>0; that is, the analysis is confined to the case with a normal, positive, Cambridge relation between profits and savings, thus

$$(c_0-c_1) + (c_1-c_2)(1-a) - c_1 b > 0 \tag{2.17}$$

In the case of a *reverse Cambridge effect* (dS/dr<0) a rise in profits and the interest rate in response to excess demand would lead to even *lower* savings and thus to a further increase in demand.[17] A reverse Cambridge effect will thus have a destabilizing effect on the income-expenditure dynamics in the medium and the short term.[18]

As can be seen from (2.17) the Cambridge effect will be positive if either $c_0>c_1$ or $c_1>c_2$ (provided that a<1) or b<0, thus if

1. capitalists have of a higher propensity to save than workers ($s_2>s_1$);
2. profits are taxed at a higher rate than wages ($\tau_2>\tau_0$);
3. interest income of workers is taxed more heavily than wage income ($\tau_1>\tau_0$);
4. the government is a net creditor (b<0).

In each of these cases a shift from wages to profits leads to an increase of aggregate saving. For the last possibility (b<0), which is of course not very likely in practice, this can explained as follows. When the state is a net creditor a rise in the interest rate leads to a shift of income from the private sector to the state. As government expenditure is assumed to be fixed, the ensuing fall in private sector consumption implies an increase in aggregate saving. Conversely, if the state is a net debtor the higher interest rate leads to a higher private sector income and thus tends to *lower* savings.[19] If this destabilizing effect of public debt is strong relative to the stabilizing impact of the simultaneous shift from wage-earners to profit-earners, this may give

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17 Note that in the present analysis the growth rate is exogenous. This assumption will be relaxed in ch.5.

18 This assumes that the interest rate varies with the profit rate according to eq. 2.12. As this equation concerns long-term equilibrium this condition is not generally true in the short period. Note that the partial effects of π and r on savings are:

$$\partial S/\partial \pi = s_2(1-\tau_2) - s_1(1-\tau_1)$$

$$\partial S/\partial r = -\{s_2(1-\tau_2) - s_1(1-\tau_1)\}a - s_1(1-\tau_1)b$$

As both π and r probably respond positively to excess demand it can de seen that the profit rate is generally stabilizing while in this model the interest has a destabilizing (positive) impact on demand. Therefore it depends on whether r or π reacts stronger if short-period equilibrium is stable or not. But even if the condition for short-period stability is less restrictive than condition 2.17, it is probable that this long-period condition will be relevant for the system's stability in the medium or the long term.

19 While the case of the state as a net creditor is not very likely, a similar effect may in the short run arise due to pension funds. It is well-known that these funds, which in practice belong to the greatest investors, are very reluctant to pass changes in their returns to their contributors. As a result a rise in property income does not lead to an equivalent rise in disposable private income, and thus to an increase in aggregate savings. This is of course only true if individuals do not look through the 'pension fund veil'.

rise to a reverse Cambridge effect. Hence public debt should not be too large, thus rewriting equation 2.17:

$$b < \frac{1}{c_1}\{(c_0-c_1) + (c_1-c_2)(1-a)\}$$

it can thus be seen that there exists a maximum for public debt beyond which the Cambridge effect becomes reversed.

Unfortunately, the model described above gives no neat solution for the steady state. Solving the differential system for Da=0 and Db=0 it yields a cubic function with three possible solutions. In order to grasp the basic features of the model we shall therefore examine two simplified cases: first, a quasi-schizophrenic Kaldorian version, where differential saving arises from differences in tax rates. And secondly, a Pasinetti version with different savings propensities for capitalists and workers. In both cases for simplicity it is assumed that the profit rate is equal to the interest rate (ϕ=0).

2.6 DIFFERENTIAL TAXING

The schizophrenic interpretation of Kaldor's savings equation by Samuelson-Modigliani and others, according to which differential saving is due to different attitudes of individuals towards wage income and profit income, is generally rejected (cf. Kaldor 1966, Pasinetti 1983). However, if one recognizes that taxes may also cause differences in effective propensities to save this version of Kaldor's model become more relevant again. As known, wages and profits are treated differently in most tax systems. If profits are taxed at a higher rate than wages there will exist a positive Cambridge relation even if workers and capitalists have the same savings propensities. In this section we will analyse a simple quasi-schizophrenic Kaldorian model with:

I. $s_1 = s_2 = s$ (uniform savings propensities)

II. $\tau_0 < \tau_1 = \tau_2$ (lower taxes on wages than on profits and interest).

Steady state

Denoting the uniform savings rate by s the model yields the following unique steady state solution.[20]

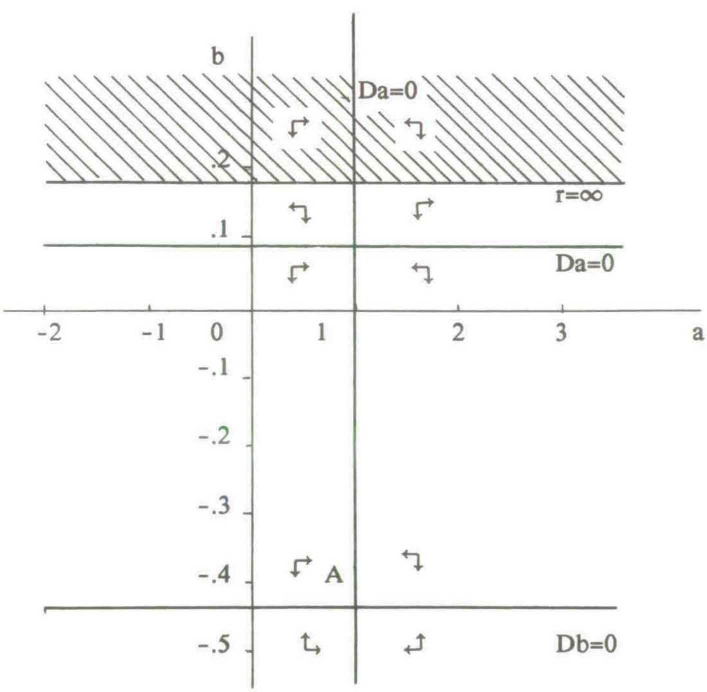
[20] The solution for Da=0 and Db=0 yields a second solution at an infinite interest rate and an undetermined debt ratio a. This solution is neglected as it makes little sense economically.

$$r = \frac{i\{i + g - \tau_0 y - s(1-\tau_0)y\}}{(1-\tau_1)i + (\tau_1-\tau_0)(1-s)i - (1-\tau_1)s(y-g)} \tag{2.18}$$
$$a = 1$$
$$b = -\frac{i - s(y-g)}{(1-s)i}$$

This solution implies the following remarkable results:
1. capitalists vanish in the long run;
2. the size of public debt is *negatively* associated with government expenditure;
3. the size of public debt is *independent* of the tax rates.

This last '*Pasinetti paradox*' *for public debt* is apparently due to the fact that the impact of the tax rates on the interest rate, and thereby on public debt, is such that it precisely offsets the direct impact of the tax rates on public debt.

Figure 2.1 Phase diagram for the quasi-schizophrenic Kaldor model



Explanation: This figure is based on the following numerical values: i=4%; y=0.4; ϕ=0; s_1=s_2=s=0.1; τ_0=0.15; τ_1=τ_2=0.2; g/y=0.16.

Dynamics

The dynamics can be discussed with reference to the phase diagram (*figure 2.1*), which shows the Da=0 and Db=0 curves in the {b,a} plane. The $r=\infty$ boundary represents the condition for a positive Cambridge effect (eq. 2.17). Only the region below this boundary is relevant. The peculiar shape of this diagram is due to the fact that, because workers and capitalists have the same net saving rate with respect to profits, Da and Db are independent of the net debt of capitalists (a). Steady state equilibrium is represented by point A. This equilibrium can be seen to be stable, locally as well as globally, for any starting point below the $r=\infty$ boundary.

The exact conditions for local stability of solution A can be derived from the linearized differential system.

$$\begin{bmatrix} Da \\ Db \end{bmatrix} = \mathbf{H}.\begin{bmatrix} a - a_s \\ b - b_s \end{bmatrix}$$

where the subscript s denotes the steady state equilibrium and the elements of the **H**-matrix are given b

$$h_{11} = -i + s(1-\tau_1)r$$

$$h_{12} = -s(1-\tau_1)(1-a)\partial r/\partial b$$

$$h_{21} = 0$$

$$h_{22} = -i + (1-\tau_1)r + \{(1-\tau_1)b - (\tau_1-\tau_0)\}\partial r/\partial b$$

where $\partial r/\partial b$ is determined by eq. 2.16. Solution A is locally stable if the real parts of the eigenvalues, evaluated in A, are negative, or by the Routh-Hurwitz conditions if

$$\text{Trace }(\mathbf{H}) = h_{11} + h_{22} < 0$$

$$\text{Det }(\mathbf{H}) = h_{11}.h_{22} - h_{12}.h_{21} > 0$$

Since $h_{21}=0$ the conditions require simply $h_{11}<0$ and $h_{22}<0$, which implies

$$i > 0 \tag{2.20}$$

$$i - s(1-\tau_1)r > 0$$

After substitution for r and taking account of the condition for a positive Cambridge effect condition (2.20) can be solved into[21]

$$i > \mathrm{MAX}\,\{\, s(1-\tau_1)y,\; s(1-\tau_1)y\,\frac{y-g}{(1-\tau_0)-s(\tau_1-\tau_0)} \,\} \tag{2.20'}$$

This condition is satisfied if the rate of investment (and government expenditure) is sufficiency high in relation to the rate of saving.

A remarkable consequence of these conditions, when they are superimposed on the solution for government debt (2.17), is that any stable equilibrium must be characterized by a *negative* government debt. As a corollary the steady state budget deficit must permanently show an *excess* of income over outlays. This is evidently odd in practice. It is apparently due to the destabilizing impact of public debt, which tends to raise expenditure, and thereby the interest rate. This gives rise to a further increase in debt service and thus of the budget deficit as well. In case of a creditor position for the government an increase in this position leads to a lower interest rate, and therefore to a smaller surplus on the budget.

2.7 DIFFERENT SAVINGS PROPENSITIES

These rather discomforting results are not specific to the Kaldor variant discussed above, but are obtained for other versions of the general model as well. As an alternative we shall now consider a Pasinetti variant where differential saving is caused by different savings propensities of workers and capitalists. This model is thus characterized by:

I. $s_2 > s_1$ (higher propensity to save of capitalists)
II. $\tau_0 = \tau_1 = \tau_2 = \tau$ (uniform tax rates)

Steady state

Under these propositions the model has two steady state solutions: a two-class 'Pasinetti' solution and a dual 'anti-Pasinetti' solution where the capitalists have

[21] Substitution of r in (2.20) yields: i>0 and

$$i\{i-s(1-\tau_1)y\}/[(1-\tau_1)i+s(1-\tau_1)(g-y)+(1-s)(\tau_1-\tau_0)i] > 0$$

As a positive Cambridge effect in the steady state requires (eqs. 2.17 and 2.18):

$$i - s(y-g) - i(1-s)(\tau_1-\tau_0)/(1-\tau_1) > 0$$

it can easily be seen that both the numerator and the denominator of the above equation must be >0, hence equation 2.20'.

disappeared. It may be noted that this second solution is 'anti-Samuelson-Modigliani' as well because income distribution is still an effective mechanism for ensuring savings-investment equilibrium, no longer indeed through redistribution between capitalists and workers but now through redistribution between workers and the state. These solutions are:

Solution (I): The Pasinetti state

$$r = \frac{i}{(1-\tau)s_2} \tag{2.21a}$$

$$b = -\frac{s_2}{1-s_2}\,\frac{g-\tau y}{i}$$

$$a = 1 - \frac{s_2}{i}\,\frac{(1-s_2)x - (1-s_1)(g-\tau y)}{(1-s_2)(s_2-s_1)}$$

Solution (II): The dual state

$$r = \frac{i}{1-\tau}\,\frac{x}{x-(1-s_1)(g-\tau y)} \tag{2.21b}$$

$$b = -\frac{x-(1-s_1)(g-\tau y)}{(1-s_1)i}$$

$$a = 1$$

where $x = i+g-\tau y-s_1(1-\tau)y$. With respect to the first solution we can observe that

1. income distribution exhibits the familiar Pasinetti feature that the interest rate (= profit rate) is determined by the growth rate and the *net* savings rate $(1-\tau)s_2$;
2. public debt is *negatively* associated with government expenditure (for any $i>0$), just as in the Kaldorian model discussed above;
3. the size of public debt is *independent* of savings of workers.

According to the dual solution, where capitalists have disappeared, the interest rate as well as the debt ratio depend on the savings propensity of workers. Further, this solution implies a normal, positive, relation between public debt and government expenditure (for any $i>0$).

Boundary conditions

These results are again subject to several boundary conditions (eq. 2.22 below). For the first (Pasinetti) solution the boundary conditions with respect to the wealth of capitalists ($a \leq 1$) and the income of workers ($y-\pi+ar \geq \chi$) are

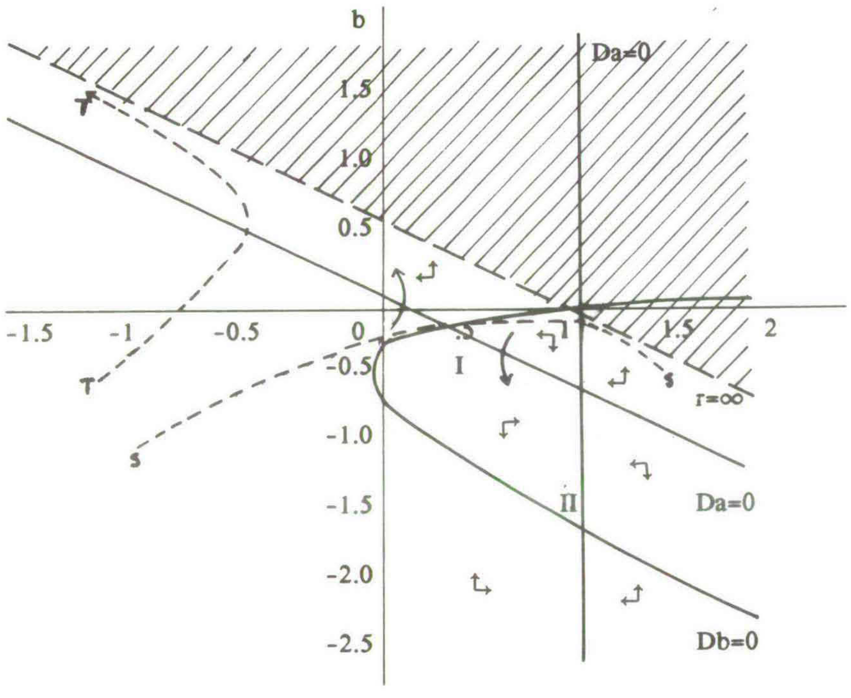
$$i-s_1(1-\tau)y-(g-\tau y) \geq 0$$

$$i-(1-\tau)\{s_1\chi+s_2(1-\chi)\} \geq 0 \qquad (2.22a)$$

respectively. These conditions are satisfied if investment is sufficiently high. For the dual solution, on the contrary, the inflation barrier ($y-\pi+ra \geq \chi$) requires investment to be low:

$$i+(g-\tau y)-(1-\tau)\{1-(1-s_1)\chi\}y \leq 0 \qquad (2.22b)$$

Figure 2.2 Phase diagram for the Pasinetti variant



Explanation: This figure is based on: y=0.4; i=0.04; τ=0.2; g/y=0.21; s_1=0.05; s_2=0.3.

Dynamics

The dynamics of this model can derived from the phase diagram (*figure 2.2*) showing the Db=0 and Da=0 conditions in the {b,a} plane. Note that Da=0 is also satisfied for any point on the a=1 line as well. The condition of a positive Cambridge effect is again limited by the {r=∞} curve; only the area below this boundary is relevant. Apart from the singularity point {a=1, b=0) the system has two solutions, one at a=0.3 (Pasinetti state) and the other at a=1 (dual state). In both cases public debt is negative. As can be seen from the diagram the second solution is locally (not globally) stable; the first solution is characterized by a saddle-point configuration and is thus unstable. As to the adjustment trajectory we can distinguish the two cases:

1. if the system starts from a point right of the separatrix S-S, but below the r=∞ boundary, it will tend to the anti-Pasinetti state (II). During this adjustment process the share of wealth of capitalists shrinks asymptotically to zero.
2. if the starting point is left of the S-S curve, public debt will grow for ever, pushing up the interest rate further and further, leading to an ever greater wealth of capitalists (path T-T in the figure).

Note that for any starting point with a positive public debt (b>0) the system can *never* reach the stable solution (II).

The conditions for local stability can be obtained from the linearized system (2.23) evaluated in its steady state solution $\{a_s,b_s\}$

$$\begin{bmatrix} Da \\ Db \end{bmatrix} = \mathbf{H}.\begin{bmatrix} a-a_s \\ b-b_s \end{bmatrix} \tag{2.23}$$

where the elements of the **H**-matrix are

$$h_{11} = -\, i + s_2(1-\tau)r - (1-\tau)(1-a)s_2\partial r/\partial a$$

$$h_{12} = -\, s_2(1-\tau)(1-a)\partial r/\partial b$$

$$h_{21} = -\, (1-\tau)b\partial r/\partial a$$

$$h_{22} = -\, i + (1-\tau)r + (1-\tau)b\partial r/\partial b$$

where $\partial r/\partial a$ and $\partial r/\partial b$ are again determined by equation 2.16. The resulting stability conditions[22] are given in *table 2.1* together with the other constraints for this model.

[22] See Appendix 2.B for the derivation of these conditions.

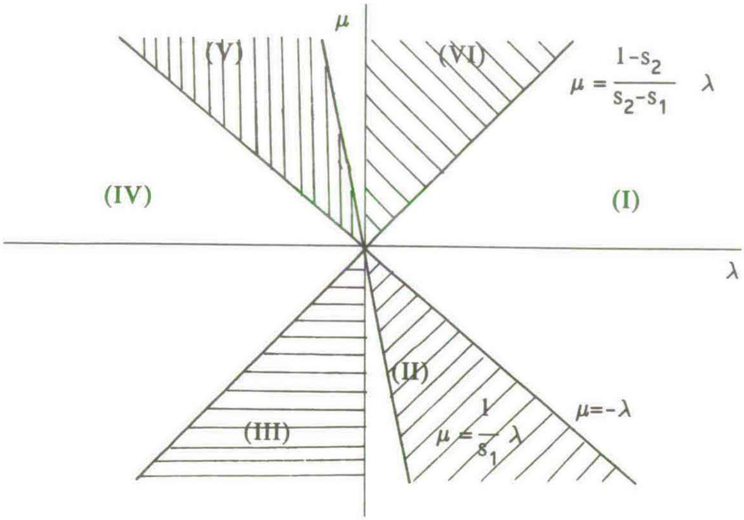
Table 2.1 Stability conditions

	Solution I Pasinetti state	Solution II Dual state
local stability:	$(\mu+\lambda)(\mu - \frac{1-s_2}{s_2-s_1}\lambda) > 0$	$(s_1\mu+\lambda)(\mu - \frac{1-s_2}{s_2-s_1}\lambda) < 0$
positive Cambridge effect:	$\mu+\lambda > 0$	$s_1\mu+\lambda > 0$
boundary conditions		
$a \leq 1$:	$\mu - \frac{1-s_2}{s_2-s_1}\lambda \leq 0$	
$y-\pi+ra>\chi$:	$\lambda-(s_2-s_1)\nu y > 0$	$\lambda+\mu-(1-s_1)\nu < 0$

where $\mu = g-\tau y$, $\lambda = i-s_1(1-\tau)y$ and $\nu = (1-\tau)(1-\chi)$

These results are illustrated graphically in *figure 2.3* which shows all possible combinations of μ and λ. For simplicity the boundary conditions with respect to the minimum wage share have been neglected.

Figure 2.3 Long-term characteristics of the Pasinetti variant



The characteristics of both solutions in each of the six regions in this graph are summarized in *table 2.2*. On the basis of these results we can conclude that:

1. there is no feasible Pasinetti state which is both stable and satisfies the boundary conditions; the first solution in each region is either characterized by negative wealth of capitalists ($1-a<0$), or by instability, or a reverse Cambridge effect.
2. in contrast, the dual solution may satisfy the stability conditions as well as the boundary conditions, but only when μ and λ lie in zone I or II, i.e. if $-\lambda/s_1<\mu<\lambda(1-s_2)/(s_2-s_1)$. These regions are however quite restrictive: if, for example, the rate of investment falls below $s_1(1-\tau)y$ the model can never satisfy these conditions. Further, note that in the stable regions I and II public debt is always negative .

Table 2.2 Characteristics of the Pasinetti model

zone	Solution I				Solution II		
	b	1-a	Cambridge effect	stability (+=stable)	b	Cambridge effect	stability (+=stable)
I	-	+	+	-	-	+	+
II	-	+	-	+	-	+	+
III	+/-	+	-	+	-	+	-
IV	+	-	-	-	+	-	+
V	+	-	+	+	+	-	+
VI	+/-	-	+	+	+	-	-

Thus, any stable long-term equilibrium, if it exists at all, is characterized by the *disappearance of capitalists* ($a=1$) and a *net creditor position of the government* ($b<0$). This conclusion is the same as the conclusion reached for the Kaldor variant above.

These unsatisfactory results in the two simplified variants considered above are not really changed for more general examples. Numerical simulations for the general model, within a fairly wide range for the parameters, did not yield any stable solution with positive government debt and positive interest rate. Apparently the 'classical' post-Keynesian model, even in its hybrid version above, is too rigid to yield satisfactory results when the government budget constraint is superimposed onto it. In this respect our analysis gives support to the Samuelson-Modigliani criticism of Pasinetti's model, that the distribution-mechanism is too restricted to ensure savings-investment equilibrium within a reasonably wide range of possible parameter settings of the model. However, we do not agree with their inference that the natural solution for these problems is to assume a variable technique of production. There exist other, more obvious, remedies. In the first place, one may introduce a more flexible savings function including interest and wealth effects. Further, one should relax the assumption of exogenous investment and also government behaviour should be modelled more carefully. Finally, one cannot truly discuss the short and long-term dynamics of private and public debt without introducing a monetary sector in the model. These latter modifications will be discussed in later chapters. By way of a preview of the consequences to be expected we shall now, in this final section of the present chapter, examine the implications for our 'classical' post-Keynesian model if a more sophisticated savings function and a variable technique of production are introduced.

2.8 INTEREST AND WEALTH EFFECTS

While maintaining the basic structure of our model we shall now introduce interest and wealth effects in the savings equation and make the technique of production dependent on the factor prices (represented by π). Again choosing linear relations the equations for the savings of workers (S_1) and capitalists (S_2) and the production technique (y)[23] become

$$S_1 = s_{1o} + s_1\{(1-\tau_0)(y-\pi)+(1-\tau_1)(a+b)r\} + s_{1r}(1-\tau_1)r - s_{1z}(a+b)$$

$$S_2 = s_{2o} + s_2(1-\tau_2)(\pi+ar) + s_{2r}(1-\tau_2)r - s_{2z}(1-a)$$

$$S = S_1 + S_2 \tag{2.1g}$$

$$y = y_o/(1-\beta.\pi) \tag{2.24}$$

while the budget constraints (2.13) and (2.14) now become

$$Da + Db = S_1 - (a+b)i \tag{2.14'}$$

$$Da = - S_2 + (1-a)i \tag{2.15'}$$

where s_{io} = autonomous saving of class i (i=1,2)
s_{ir} = interest effect on saving of class i
s_{iz} = wealth effect on saving of class i

[23] Note that y is the reciprocal of the capital output ratio (K/Y) for which it is assumed that $K/Y = (1-\beta.\pi)/y_o$.

All other equations remain the same. For the steady state the model again gives three possible solutions. Unfortunately the price of greater flexibility of the model is, as always, a loss of transparency.

As it is not rewarding to examine this model analytically we shall present some numerical exercises. In general, it has become clear from the numerical simulations that this model yields 'normal' solutions for a wide range of parameter settings. This is apparently due to the mitigating influence of interest effects on saving and the choice of technique. However, inclusion of the wealth effects clearly proved to worsen the stability of the model.

Dynamics

Figure 2.4a shows the phase diagram for a 'Pasinettian' set of parameters with uniform tax rates and different saving rates for workers and capitalists. As the figure brings out this model has two solutions in the relevant region where $a<1$, one of which is stable and the other is not. The stable solution (A) is characterized by a positive public debt ($b/y=1.39$) and a positive (real) interest rate ($r=1.4\%$). Moreover, this solution satisfies the boundary conditions for any reasonable minimum for the income of workers.

Whether the system actually tends to this stable solution depends on its initial position. Particularly if the system starts with a large initial public debt it may develop into a unstable process of continuously growing public debt and falling debt of capitalists (the C-C path in the figure). If public debt is even so large that the starting point is above the $\{r=\infty\}$ boundary the system is subject to a reverse Cambridge effect as well.

Adjustment process

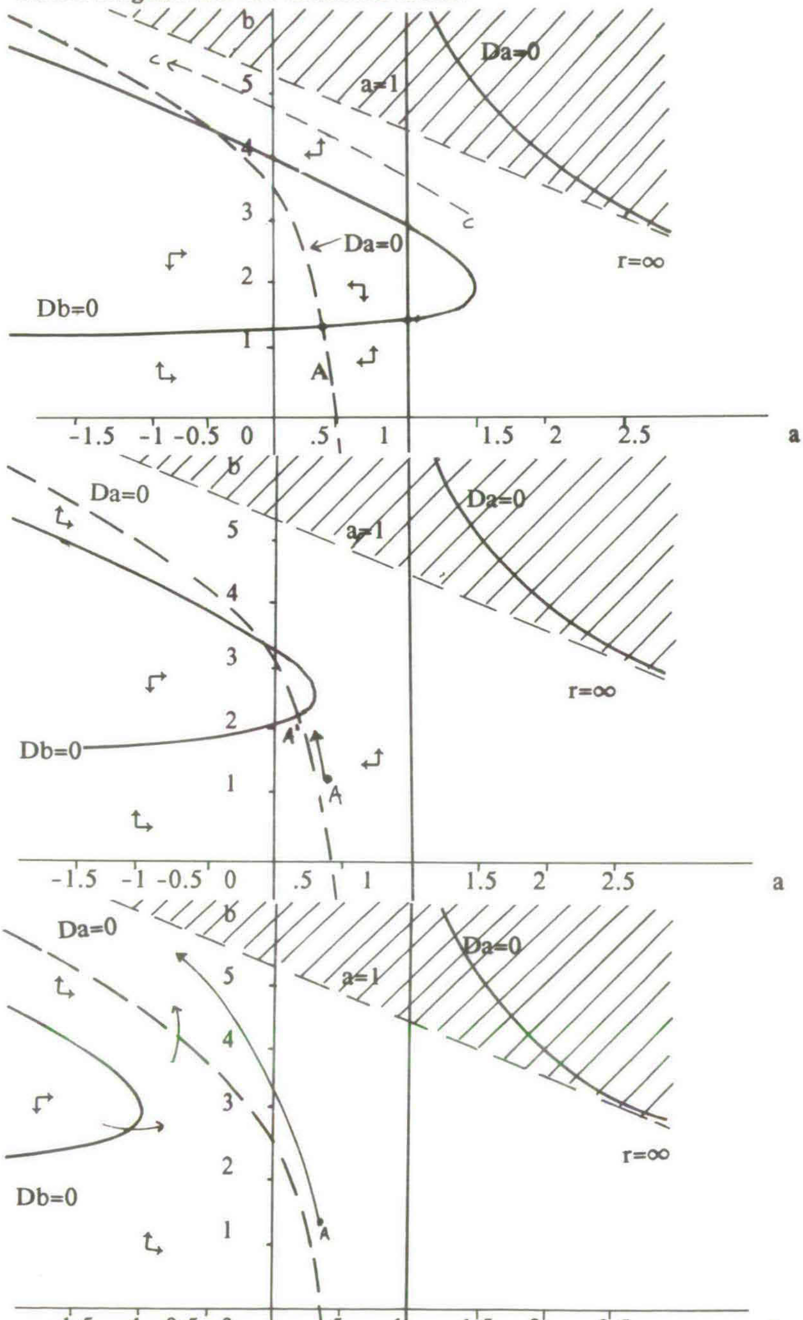
Now imagine that the system is in its stable equilibrium (A) and that government expenditure is raised from 0.24 to 0.25. As *figure 2.4b* brings out the system will tend gradually to the new equilibrium (A') with a higher b and lower a. If, however, government expenditure is raised further to 0.26, the stable solution disappears and the system becomes entirely unstable, whatever its initial position. This is shown is *figure 2.4c*.

Figure 2.4 Phase diagram for the modified model

Fig. a

Fig. b

Fig. c



Explanation: i=4%; s_1=0.1; s_2=0.4; $\tau_0=\tau_1=\tau_2=0.2$; y_o=0.4; β=0; ϕ=1; $s_{1r}=s_{2r}$=2; s_{1o},s_{2o},s_{1z},s_{2z}=0; g/y=0.24, 0.25, 0.26

Figure 2.5 The adjustment process

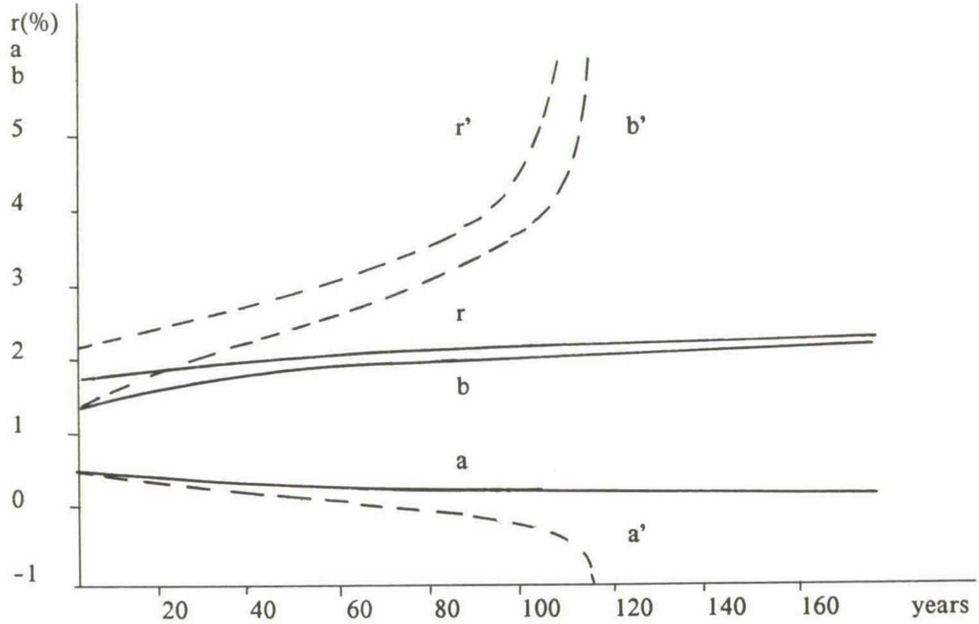


Figure 2.5 shows the adjustment trajectories following the rise in government expenditure from 0.24 to 0.25 and 0.26 respectively on a time axis. If government expenditure is raised to 0.25 the simulated time path shows a slow and gradual rise in public debt (b) and the interest rate (r) and a fall in debt of capitalists (a) to their new steady state values. This is shown by the solid curves. In the second case, when g is raised to 0.26 the adjustment process (dashed line) also happens to be as slow and gradual for many years. However, after several decades this gradual process changes quite abruptly into a accelerating process with a sharply rising interest rate and a polarizing distribution of wealth.

Minimum growth rate

These exercises bring out that the system yields a stable steady state equilibrium as long as g does not exceed a certain limit. Similarly, one can establish a boundary below which the growth rate i should not fall for a stable equilibrium to exist. For example if g=0.25 there is a stable solution as long as i>3.4%. Thus for i=4%, as in the example above, it does indeed possess a stable solution. However, when g=0.26 the critical value for i rises to 6.3%, which exceeds the growth rate in our example, so that the system becomes inevitably unstable.[24]

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[24] In chapter 6 it will be shown that this critical growth rate corresponds to the bifurcation point of a catastrophe manifold.

Table 2.3 Partial effects on the minimum growth rate necessary for stability

parameter	effect on i	parameter	effect on i
τ_0	-3.66	s_{1r}*)	-3.99
τ_1	-0.18	s_{2r}*)	-4.73
τ_2	-0.32	s_{1z}	4.68
s_1	-1.75	s_{2z}	6.06
s_2	-0.30	g	4.35
s_{1o}	-2.36	β	-0.002
s_{2o}	-2.81	ϕ*)	-6.00

Explanation: These effects have been calculated with reference to the steady state given in figure 2.4. One should be careful to compare the magnitude of these effects as an equal change in each variable may entail very different impacts in absolute amounts.
*) In order to make these effects more comparable to the other effects they have been multiplied by 100 as they are attached to the interest or growth rate.

Table 2.3 gives the partial effects of each parameter on the growth rate which is at the minimum required for a stable solution to exist. This partial effect is thus a measure for the stabilizing or destabilizing impact of those parameters. If a parameter has a stabilizing influence, an increase in this parameter will reduce the critical growth rate; if a parameter has a destabilizing impact it raises the minimum growth rate. The results corroborate our observations above. Government expenditure (g) and the wealth effects (s_{1z}, s_{2z}) turn out to have a strong destabilizing impact on the system. All other variables appear to mitigate the intrinsic instability of the system, including the substitution coefficient of production technique (β).

The destabilizing impact of the wealth effect on saving is remarkable as in most other investigations of the dynamics implied by the government budget constraint it proved to be an essential stabilizing factor (cf. Blinder and Solow 1973, Tobin and Buiter 1976, Christ 1979, Rau 1985 and also Asada 1987). This contrary result is a consequence of our focus on long-term growth equilibrium. Most other investigations have concentrated on a short-term Keynesian world with price rigidity and without growth. We shall return to these matters in later chapters, when the medium-period dynamics, so far neglected in our model, are explicitly taken into account.

2.9 CONCLUSION

In this chapter we have examined the dynamics of long-term asset accumulation on the basis of a simple two-class post-Keynesian model including a government sector. This model has been built on two central relationships: the relation between income distribution and aggregate saving which ensures saving-investment equilibrium, and the relation between the budget constraints and the growth of wealth (or debt) which determines the long-term dynamics of the model.

Our discussion of differential saving revealed that the mere existence of retained earnings is not a sufficient explanation of a higher propensity to save out of profits than out of wages. Therefore, we developed a synthesis between Kaldor's view that differences in savings propensities must be explained from the nature of business income, and Pasinetti's standpoint that savings propensities should be attached to social groups or classes. The latter implies that one may not neglect the role of accumulating financial wealth or debt of firms and their owners.

Introduction of the government budget constraint gave support to the Samuelson-Modigliani (1966) argument that the post-Keynesian model is too rigid to yield acceptable solutions for long-term equilibrium. It was found that a stable solution, if it exists at all, is always characterized by the disappearance of one class (the 'capitalists') and by a negative public debt. This is, of course, not very likely in practice.

In the final section it was shown that this counter-intuitive result may be remedied by introducing interest sensitivity of savings. In this respect we followed a different route from Samuelson and Modigliani who argued that the limitations of the post-Keynesian model should be solved by introducing a well-behaved production function. The numerical exercises in our final section indicated that a flexible technique of production may indeed mitigate the intrinsic instability of the system, but that it is by no means a necessary, or even the most important factor.

The model considered so far is too simple to give an appropriate account of the dynamics of public debt in a two-class model. The assumption of an exogenous growth rate and the neglect of the monetary sector is especially unsatisfactory. In the ensuing chapters we shall therefore develop a more sophisticated model with endogenous investment and a proper representation of medium-term dynamics.

Appendix 2.A A life-cycle model of differential saving

This appendix shows that a life-cycle model of saving can be fully consistent with the Cambridge savings equation if two distinct classes (workers and capitalists) are introduced with different attitudes towards saving and bequests. The model is a simple overlapping generations model with two classes (workers and capitalists) and two generations of each class (young and old) living at any one time. Following Pasinetti 1983 we assume that workers save only for their old age, whereas capitalists save for inheritance too. Choosing suitable dimensions the budget constraints for each group can be written as:

young workers: $W = C_{wy} + Z_{wo}(+1)$

pensioned workers: $(1+r)Z_{wo}/(1+n) = C_{wo}/(1+n)$

young capitalists: $(1+r)Z_{cy} = C_{cy} + Z_{co}(+1)$

old capitalists: $(1+r)Z_{co}/(1+n) = C_{co}/(1+n) + Z_{cy}$

where C_{ij} = consumption of group i,j
Z_{ij} = wealth of group i,j (beginning of period)
n = population growth
r = profit rate or interest rate
W = wage
i = w,c for workers and capitalists respectively
j = y,o for the young and the old respectively
$X(+1)$ = X in the next period

Young workers receive wage-income only. Pensioned workers consume the whole of their capital saved in the foregoing period. Young capitalists receive an inheritance at the beginning of the period. The amount of this inheritance is decided by the pensioned capitalists. For simplicity the rate of interest is assumed to be equal to the profit rate (no entrepreneurial reward). Further, employment is assumed to be given by the number of young workers. Capital stock is determined by savings in the past. Choosing capital stock equal to unity we obtain:

$$Y = W + r$$

$$1 = (Z_{wo} + Z_{co})/(1+n)$$

Now define with respect to consumption behaviour:

$$C_{wy} = c_w W$$

$$C_{cy} = c_1(1+r)Z_{cy}$$

$$C_{co} = c_2(1+r)Z_{co}$$

The consumption rates c_w, c_1 and c_2 are determined by the intertemporal optimum for each group. The optimization procedure is well-known in literature (cf. Baranzini 1982) and does not need to be repeated here.

Let us first consider the short-term solution for aggregate consumption C

$$C = c_w W + (1+r)Z_{wo}/(1+n) + c_1(1+r)Z_{cy} + c_2(1+r)Z_{co}/(1+n)$$

After substitution for W, Z_{co} and Z_{cy} we get

$$C = c_w(Y-r) + c_c(1+r) + (1+r)(1-c_c)Z_{wo}/(1+n)$$

where $c_c = c_1(1-c_2)+c_2$ is the average consumption rate of the capitalist class (young and old). Since Z_{wo} is predetermined, the effect of a shift in income distribution in favour of profits (r) is given by

$$dC/dr = -(c_w-c_c) + (1-c_c)Z_{wo}/(1+n) + X_r$$

where the first two terms on the right hand side represent the distribution effect of a change in r, and X_r measures the intertemporal substitution effect (the effect on c_w and c_c). Note that the distribution effect is negative, i.e. a normal *Cambridge effect* of profits on savings, if, and only if

$$c_w > c_c + (1-c_c)Z_{wo}/(1+n)$$

The overall effect depends on both the distribution effect and the substitution effect (X_r), the outcome of which is not certain a priori. It is evident that this condition is more likely to be fulfilled if the share of workers in total wealth is smaller, and thus the income of pensioned workers is less relative to the income of capitalists. In the extreme case with $Z_{wo}=0$ this condition reduces to the familiar Cambridge condition $c_w>c_c$. However, for the other extreme, i.e. a single class model without capitalists (therefore $Z_{wo}/(1+n)=1$), the distribution effect is certainly negative, entailing a *reverse Cambridge effect*, for any $c_w<1$. Hence we can conclude that for a normal Cambridge effect workers should own not too large a share of total wealth.

Now consider the long run. In steady state equilibrium with constant n, r and W the amount of wealth per person must be constant as well, hence

$$Z_{wo} = Z_{wo}(+1) \quad \text{and} \quad Z_{co} = Z_{co}(+1)$$

Substitution in the budget constraints for workers and capitalists yields

$$Z_{wo} = (1-c_w)W$$

$$1+r = (1+n)/(1-c_c)$$

This latter result is remarkable as it implies that this model is also subject to the *Pasinetti paradox*; that is, the profit rate depends exclusively on the growth rate and the savings propensity of capitalists. Again, this conclusion is of course valid only for $Z_{co}>0$; if $Z_{co}=0$ this model yields an anti-Pasinetti dual, just as in the more traditional

Pasinetti models.

If these results are substituted in dC/dr above we obtain

$$dC/dr= -(1-c_c)\{n+c_w-(1-c_w)Y\}/(1+n) + X_r$$

and hence a normal Cambridge distribution effect (dC/dr<0) if

$$n >(1-c_w)Y - c_w$$

The rate of growth (and thus the rate of investment) should thus be sufficiently large in relation to the savings of workers. This condition is not surprising as the share of workers in total wealth is less as they provide less of the savings necessary for investment.

Appendix 2.B Dynamics of the Pasinetti variant of the model with government.

This appendix determines the conditions for local stability of the Pasinetti-variant of the model with a government sector in section 2.7.

Solution I

Linearization of (2.14) and (2.15) gives

$$\begin{bmatrix} Da \\ Db \end{bmatrix} = \mathbf{H}.\begin{bmatrix} a-a_s \\ b-b_s \end{bmatrix}$$

After substitution for r the coefficients of the **H** matrix are

$$h_{11} = -s_2(1-\tau)(1-a)\partial r/\partial a$$

$$h_{12} = -s_2(1-\tau)(1-a)\partial r/\partial b$$

$$h_{21} = (1-\tau)b\partial r/\partial a$$

$$h_{22} = (1-s_2)i/s_2 + (1-\tau)b\partial r/\partial b$$

and the Routh-Hurwitz conditions

$$RH(1): -s_2(1-\tau)(1-a)\partial r/\partial a + (1-s_2)i/s_2 + (1-\tau)b\partial r/\partial b < 0$$

$$RH(2): -(1-s_2)(1-\tau)(1-a)\partial r/\partial a > 0$$

After substitution for a and ∂r/∂a the second condition yields

$$\{(1-s_2)x - (1-s_1)(g-\tau y)\}/x > 0$$

where $x = i+g-\tau y-s_1(1-\tau)y$.
Now consider RH(1). After substitution for a, b, $\partial r/\partial a$ and $\partial r/\partial b$ we find

$$i - \frac{i}{s_2}\frac{(1-s_2)x - (1-s_1)(g-\tau y)}{x} > 0$$

Now define $\nu = g-\tau y$ and $\mu = i-s_1(1-\tau)y$ which implies $x = \nu+\mu$. It can then be assessed that these conditions together require

$$(\nu+\mu).(\nu - \frac{1-s_2}{s_2-s_1}\mu) > 0$$

Solution II
For the second solution where a=1 we find $h_{12}=0$ which reduces the Routh-Hurwitz conditions to $h_{11}>0$ and $h_{22}>0$. After substitution for r and $\partial r/\partial b$ these conditions yield

$$i > 0 \quad \text{and} \quad \frac{(1-s_2)x - (1-s_1)(1-\tau)y}{x - (1-s_1)(g-\tau y)} > 0$$

which is satisfied if in terms of ν and μ defined above

$$i > 0 \quad \text{and} \quad (s_1\nu+\mu)(\nu - \frac{1-s_2}{s_2-s_1}\mu) < 0$$

These results are presented in *table 2.1* in the text.

CHAPTER 3

FINANCE, RISK AND THE GROWTH OF THE FIRM

3.1 INTRODUCTION

One of the distinctive features of post-Keynesian theory is the proposition that the profit rate is determined by the growth rate rather than vice versa (cf. Kaldor 1966, Asimakopoulos 1986). This theorem is based on the differential savings equation and the Keynesian notion that, in the aggregate, savings are determined by investment. In criticizing this theorem Samuelson and Modigliani (1966) pointed out that the alleged growth rate - profit rate relation is merely an equilibrium condition which must be satisfied in every model of growth, and is thus by no means exclusive to post-Keynesian theory. A similar view was put forward by Brems (1979) who argues that, formally, in any theory the growth rate and the profit rate are determined simultaneously. The difference between post-Keynesian and other theories should therefore be motivated in terms of the relative independence of investment from savings and the responsiveness of income distribution to discrepancies between ex ante savings and investment.

Post-Keynesian authors have, however, proved very reluctant to give an appropriate account of these relationships. This is especially true for the explanation of investment. It is striking that for this variable, which is assigned such a central place in the explanation of growth and distribution, hardly even the outlines of a coherent theory among post-Keynesian authors can be distinguished. Joan Robinson (1956, p.244) even explicitly refuses to develop a theory of investment: "there is no way of reducing the complexities of the inducement to invest to a simple formula. We must be content with the conclusion that, over the long run, the rate of accumulation is likely to be whatever it is likely to be". In a similar vein Asimakopoulos (1986,p.89) argues: "The possible relations between finance, investment and saving in the post-Keynesian approach are thus complex [...]. No general statement about their relationship which does not recognize (the particular historical) circumstances can adequately represent the post-Keynesian position." This opinion, although understandable, is unsatisfactory as it frustrates the further development of post-Keynesian theory.[1]

1 Robinson (1962, p.37) seems aware of this problem herself as in her 'model of accumulation' she proposes to express the 'animal spirits' as a function of expected profits.

This chapter develops a coherent microeconomic framework for the long-term explanation of investment. Our starting point is the growth strategy of a representative, permanently growing firm. The central question of the present analysis is what growth rate a firm will choose when it takes full account of the consequences of sustaining this growth rate for its financial position and its risk posture in the long run. This setting of the problem is basically similar to that of the so-called 'managerial' theories of growth of the firm (cf. Williamson 1966, Uzawa 1969, Marris 1971, Solow 1971, Odagiri 1981). The managerial approach is especially interesting because it takes account of the conflict of interests between the managers and the owners of the firm (shareholders). However, these theories concentrate on the organizational and product market constraints on the firm's growth and tend to neglect the financial limitations.

This chapter is organized as follows. After a concise overview of post-Keynesian and 'managerial' theories of investment (section 3.2) and a brief discussion of the financial limitations to growth (section 3.3), we shall in section 3.4 develop a basic model for a small (corporate) firm without access to the equity market. It will be shown that this firm faces a trade-off between the rate of growth and its risk posture (section 3.5). Section 3.6 establishes the optimum growth rate on the basis of this growth-risk frontier and the managerial preferences towards growth, risk and profitability. Finally, section 3.7 considers the determination of the pay-out of profits to shareholders. In the next chapter this basic model will be extended by the introduction of the equity market and the costs of growth. Attention will then also be paid to the adjustment process.

3.2 POST-KEYNESIAN THEORIES OF INVESTMENT

Although a well-developed post-Keynesian theory of investment does not exist, it is possible to distinguish three basic approaches:

a. the 'internal savings' approach, which links investment to the flow of retained profits;
b. the 'investment opportunities' approach, which concentrates on the limited availability of profitable investment projects;
c. the 'managerial' approach, which explains the restraint on investment from the organizational and marketing efforts necessary to sustain a certain rate of growth.

a. Internal savings

The first approach builds on the classical and Marxian 'surplus' theories of saving and investment, according to which accumulation takes place primarily through reinvestment of current profits. This idea was taken up by Kalecki (1937,1943) and later elaborated by Wood (1975) and Eichner (1976). A very strict variant is found in

Pasinetti (1981) who assumes that growth in each sector is financed purely by internal savings. Kalecki (1937,1943) allows for the possibility of debt financing in addition to internal savings. However, by assuming that firms aim at a constant rate of indebtedness, Kalecki also finds a fixed relation between investment and internal savings. If investment is raised above this target rate, more external funds are needed. As a result the rate of indebtedness rises, and thus the risk increases. On the basis of this '*principle of increasing risk*' Kalecki (1937, p.447) proposed the following expression for investment:

$$I = (1-d)S + V$$

where I = investment, S = internal savings, d = ratio of debt to net worth and V measures the effect of a shift in the marginal rate of return schedule.

A more sophisticated model is given by Wood (1975), who establishes a 'finance frontier' for corporate firms, representing the maximum rate of investment given the net returns of the firm and its targets for the pay-out of net returns (θ) and the degree of external financing (ϵ), thus[2]

$$I \leq \frac{1-\theta}{1-\epsilon}(\pi - a.r)K$$

where π = profit rate, r = interest rate, a = ratio of debt to capital stock, ϵ = fraction of investment financed externally, and K = capital stock. The main weakness of this model, and of Kalecki's, is that the target variables θ and ϵ, are assumed to be fixed. Although Wood is right that in the short term firms tend to rely on given norms for their financial policy, it seems unwarranted to consider these norms as exogenous in the long run too. In our view a long-term analysis should explain why a firm chooses a particular level for its targets. Therefore Wood's analysis appears to be suited more to the short or medium term than for the long term.

b. Investment opportunities

The second approach takes just the opposite point of departure and concentrates on investment-opportunities rather than finance as the limiting factor of the firm's growth. Following Keynes it is assumed that the evolution of a well-developed financial system has separated the investment decision from the ex ante saving and finance decisions. As is well-known, this 'separability' theorem was later formulated rigorously by Modigliani and Miller (1958), who showed that the method of finance is irrelevant to the cost of capital and therefore to the investment decision as well.

The focus in (post-)Keynesian theories of investment thus shifted to investment

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2 For expository reasons we have neglected depreciation.

opportunities as the basic limiting factor. Financial aspects received little attention, being reduced to a given (exogenous) cost of capital (cf. Harrod 1948), treated as a fixed side condition (cf. Kaldor 1961, Kaldor and Mirrlees 1962), or neglected altogether (cf. Robinson 1956, 1962).[3] Instead, investment is related - in an ad-hoc manner - to demand (Harrod 1948), prospective profits (Robinson 1963, Kaldor and Mirrlees 1962) or a combination of both (Kaldor 1957,1961).[4]

c. Managerial theories of growth

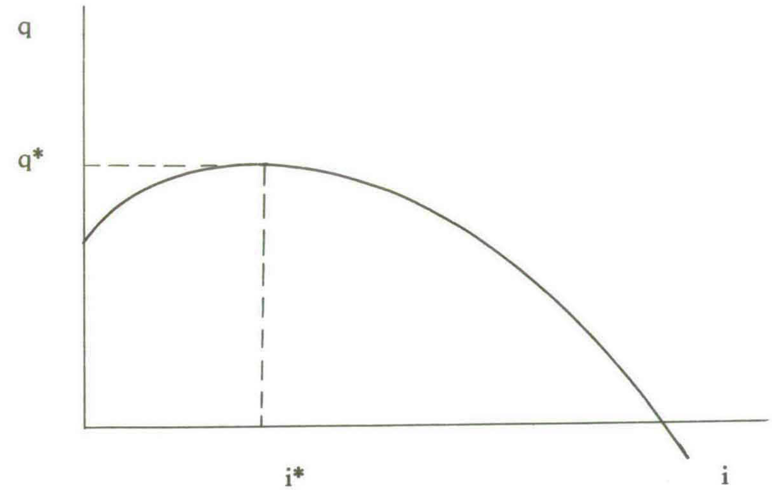
Managerial, or 'corporate,' theories of growth[5] concentrate on the dynamic constraints on the growth of individual (corporate) firms due to the managerial and marketing efforts necessary to keep up a certain growth rate. In contrast with the foregoing approaches this theory is thus essentially microeconomic. The core of the theory of corporate growth is a concave '*growth-valuation*' frontier representing the trade-off between the valuation of the firm (q) and its growth rate (i) (*figure 3.1*). Because shareholders desire maximization of the market value of their shares they would prefer growth rate (i^*) in the figure which corresponds to the highest possible valuation (q^*). However, as managers are more interested in the expansion of the firm, they will generally select a higher growth rate ($i>i^*$). Given the concavity of the q-i frontier the actual growth rate will depend on the preferences of the management and on their discretionary power vis-à-vis the shareholders.

3 According to Robinson (1956, p.51) internal finance may hold back investment at best temporarily : "It is a large and rapid rise in the rate of investment, not a high rate of investment which the finance limit prevents."

4 In his early growth models Kaldor (1957,1961) divides investment into a demand related ('acceleration') component and a profit related component. Kaldor motivates the latter component with reference to Kalecki's principle of increasing risk. He does not elaborate this financial aspect further. In his 'new' model (Kaldor and Mirrlees 1962) Kaldor introduces his well-known 'pay-back period' criterion for investment; this 'rule of the thumb' implies that investment is related to the (undiscounted) prospective returns within a certain time-horizon. Recently Blatt (1983) has shown that in the presence of 'Knightian' uncertainty this criterion may be warranted in the short term when firms rely on given norms and targets. However, in the long run the length of the pay-back period should be considered as variable and should be explained in terms of the risks and the costs of failure of investment projects. Hence even in this approach one cannot explain long term investment behaviour without an assessment of the financial position and risk posture of the firm (see also footnote 13 below).

5 Seminal contributions to the managerial theory of growth have been made by Baumol (1959), Penrose (1959), Marris (1964) and Williamson (1966). Although these contributions are certainly non-neoclassical, they are not usually labelled as post-Keynesian. Nevertheless we have included them in our survey of post-Keynesian investment theory because this approach has much in common with the post-Keynesian approach, especially the emphasis on institutional and behavioural factors. Furthermore several post-Keynesian authors have adopted this approach in order provide a microeconomic foundation for the traditionally macroeconomic oriented post-Keynesian theory (cf. Eatwell 1971, Wood 1971, 1975, Eichner 1975, 1983).

Figure 3.1 Growth-valuation frontier



For the motivation of the q-i frontier two approaches can be distinguished, one emphasizing the internal, managerial costs of growth, and the other concentrating on the external, marketing costs of growth.

1. According to the *internal* approach the growth of firms is constrained by the limited capacity of the management and the time and effort necessary to find, train and absorb new managers. Sometimes this constraint is conceived as an absolute limit on the capacity to grow (cf. Penrose 1959, Slater 1980, Moss 1984), while other authors assume managerial costs to rise or the efficiency to fall as the firm expands faster (cf. Williamson 1966, Uzawa 1969, Baker 1978, Odagiri 1981).
2. The *external* approach explains the negative growth-profitability relation by the costs for advertisement and R&D necessary to shift the demand curve for the firm's products. This idea, which was put forward by Marris (1964), has been elaborated by Solow (1971), Auberada (1979) and Seoka (1985). These authors include the 'stock of goodwill' in the demand function for the firm's products and explain the growth of goodwill from the amount of marketing effort. Lintner (1971) extended Marris' model to an uncertain environment, and also showed that a positive association between growth and the variability of profits may be sufficient to impose an effective constraint to the firm's growth rate.

These models of corporate growth provide an interesting microeconomic foundation of investment behaviour. However, as in the 'investment opportunities' approach discussed above, they concentrate on the real costs of growth and pay little attention to the financial aspects. Managerial theories appear to follow the separability approach of finance and investment too. This is unsatisfactory, particularly for a theory which stresses market imperfections and conflicting interests between shareholders and managers.[6]

3.3 FINANCIAL LIMITATIONS TO GROWTH

In contrast to the 1960's and 1970's during which the Modigliani-Miller 'irrelevance of finance' view reigned, it is nowadays widely recognized that the financing decision is an essential element of the firm's development strategy as a whole. In this connection it may be noticed that the focus of financial theory has shifted from market equilibrium to the finance decisions of individual firms.[7] It is explicitly recognized now that the firm is the typical legal and organizational entity for production and investment; investment projects do not exist on their own, they exist only when embedded in an organizational entity.[8]

As a corollary the risk of financing investment projects is not attached to these projects as such but to the firm which carries them out. It is thus natural for capital market investors and financial intermediaries to base their lending decisions on the creditworthiness of the firm rather than on the direct merits of the investment plan. Only in a perfect Modigliani-Miller world without liquidity constraints, irreversibilities, information costs, (re)organization costs and costs of bankruptcy, would it be permissible to abstract from the organizational and legal structure. However, in such a world firms would not exist either and every investor would run his/her own project.

One important aspect of the imperfection of financial markets is that the opportunities for risk sharing are limited. Due to informational imperfections (most of them related to the intrinsically asymmetrical nature of information about firms)

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6 It may even be questioned if the separability theorem is not inconsistent with the idea of conflicting interests between managers and shareholders. As is known, this separability theorem is valid only in a Modigliani-Miller world with perfect capital markets and in the absence of bankruptcy costs and tax subsidies. It is, however, difficult to see how with perfect capital markets the firm's strategy can ever diverge from the interests of shareholders; any discrepancy between the actual and the potential maximum would immediately lead to intervention by the shareholders. Since Jensen and Meckling's (1976) seminal contribution on agency theory it is now widely accepted that once the assumption of 100% control by the shareholders is relaxed, one must also take account of the consequences of the finance decision for the 'ownership structure' of the firm and hence for the cost of controlling the management ('agency costs').

7 In their introduction to a recent special issue on corporate finance of the Journal of Financial Economics, Jensen and Warner (1988, p.19) report "the expansion of financial economists' interests from financial markets to research on behavior within corporations."

8 With respect to the modern theory of finance some authors make a distinction between agency theory, and transaction cost theory; agency theory conceives the firm as a nexus of contracts, whereas the transaction cost approach concentrates on the governance of production and finance (cf. Williamson 1988).

the equity market is far from perfect; some authors even consider equity typically to be rationed (cf. Greenwald and Stiglitz 1988a, 1988b). Consequentially, if firms have only limited recourse to issues of new shares, they must manage risks on their own. This implies that when deciding on production and investment they should take good account of the consequences of these decisions for the financial position and risk posture of the firm.

Further, the recognition of firms as distinct organizational entities raises the issue of diverging interests between those who control the firm (the managers) and those who supply the finance to it (shareholders, bondholders, credit institutions). In their pioneering article on the agency theory of the corporate firm Jensen and Meckling (1976), showed that the divergence of interests between the 'inside owners' of the firm (the managers), and the 'outside owners' (the investors without a direct role in the management) generates *agency costs*. These costs consist of the efforts of bonding and monitoring and the residual loss to the investors (the 'principals') because the managers (the 'agents') pursue different interests from theirs. According to Jensen and Meckling (1976, p.312) managers are more interested in non-pecuniary benefits, such as "the physical appointments of the office, the attractiveness of the secretarial staff, a larger than optimal computer to play with,... etc," than in the pecuniary benefits which are reflected in the present value of the firm alone. As a result inside owners (the managers) aim at a lower efficiency, and thus a lower valuation than outside shareholders.

With respect to finance the critical distinction in Jensen and Meckling's theory is thus not between the *types* of finance - debt or equity - , but between the *sources* of finance: *inside* finance (from the personal wealth of those who control the firm) or *outside* finance (from investors without any actual control).[9] A basic proposition of their analysis is that as the volume of outside finance increases relative to inside finance the divergence in interests will grow, thus leading to higher agency costs. This applies to debt as well as to equity finance.

It is obvious that these theories on imperfect financial markets give new support to the 'internal finance' approach of investment. In the presence of external financial constraints the amount of inside finance, i.e. the wealth or savings of the inside owners, is, of course, the essential limiting factor to the expansion of the firm.

In this chapter we shall develop a model of the growth of the firm concentrating on the financial constraints. From the agency theory we adopt the proposition that the supply of internal finance is a principal limiting factor to the growth of the firm. Attention will also be given to the conflict of interests between managers and shareholders. We do not follow Jensen and Meckling, however, in their neglect of risk.

9 In their theory the optimum debt-equity mix of finance is in fact a secondary problem, following from the structure of agency costs which are different for debt and equity.

In this respect we agree with Greenwald and Stiglitz (1988b, p.252) that "every production decision is a risk decision." Just because of the limited supply of inside finance it can be shown that there is a crucial relationship between the rate of expansion and the risk posture of the firm. In essence, this relationship is similar to Kalecki's principle of increasing risk.

3.4 A BASIC MODEL

In order to concentrate on the financial aspects we shall consider the following elementary model of a (small) corporate firm. The firm is a price-taker on both the input and the output markets. Its production is subject to constant returns to scale and diminishing marginal productivities of capital and labour. Production factors can be adjusted instantaneously and without cost. As far as the firm holds positive stocks of financial assets (including bank deposits) these yield the same interest rate as the firm's debt. As we wish to concentrate on the internal constraints on growth, the supply of loans is assumed to be perfectly elastic at the given interest rate; there exist no liquidity constraints and no credit rationing. Finally, it is, as a first step, assumed that the firm has no access to the equity market at all; it is thus fully equity rationed.

The firm operates in a 'steady state' environment: Technical change is purely labour augmenting and (real) wage growth is equal to productivity growth, so that the profit rate and relative prices are constant. The interest rate, time preference etc. are constant as well.

The model uses the following variables (all stock and flow variables are in real terms and expressed as ratios to capital stock):

a = debt, net of holdings of financial assets
i = net investment (= growth rate of capital stock)
l = labour employed (in efficiency units)
p = rate of inflation
P = output price
r = real interest rate (net of taxes)
R = nominal interest rate (= r+p)
W = real wage rate (per efficiency unit)
y = production (value added)
δ = pay-out of profits to shareholders (dividends)
π = profit rate (net of taxes)
τ_π = tax rate on profits
τ_r = tax rate on nominal interest payments
τ_p = tax rate on inflationary change in nominal debt
ψ = depreciation rate

The relevant relations for the firm may be then modelled as

$$Da = -y + Wl + Ra + \delta + (i+\psi) + \tau_\pi(y-Wl-\psi) - (\tau_r R-\tau_p p)a - a(i+p) \tag{3.1}$$

$$\pi = (1-\tau_\pi)(y-Wl-\psi) \tag{3.2}$$

$$r = (1-\tau_r)R - (1-\tau_p)p \tag{3.3}$$

$$y = y(l) \qquad y>0;\ y'>0\ ;\ y''<0 \tag{3.4}$$

Equation 3.1 represents the budget constraint which states that the change in debt Da is equal to the payments of wages Wl, interest Ra, pay-out of profits to shareholders δ, gross investment $(i+\psi)$ and taxes on profits and interest payments less income from production y and less the impact of nominal growth on the debt ratio, a(i+p). The second and third equations define the profit rate π (after taxes) and the real interest rate r (after taxes). Equation (3.4) represents the linear homogeneous production function with the usual assumptions on differentiability.

Then, after substitution of equations (3.2) and (3.3) the budget constraint (3.1) becomes

$$Da= -(\pi-\delta) + ra + (1-a)i \tag{3.5}$$

This equation implies that in the steady state (Da=0) the debt ratio is determined by the growth rate, the profit rate net of pay-out to shareholders and the interest rate

$$a = \frac{i-(\pi-\delta)}{i-r} \tag{3.6}$$

Confining our argument for the moment to the case where retained profits exceed the interest rate $(\pi-\delta)>r$ and $i>r$[10] we can distinguish the following cases:

net creditor:	$a < 0$	if $(\pi-\delta) > i > r$
net debtor:	$0 < a < 1$	if $i > (\pi-\delta) > r$
insolvency:	$a > 1$	if $i > r > (\pi-\delta)$

These results are obvious: if retained profits persistently exceed investment the firm becomes a net creditor in the long run. If retained profits are less than needed for investment the firm must continuously raise external funds and will thus become a net

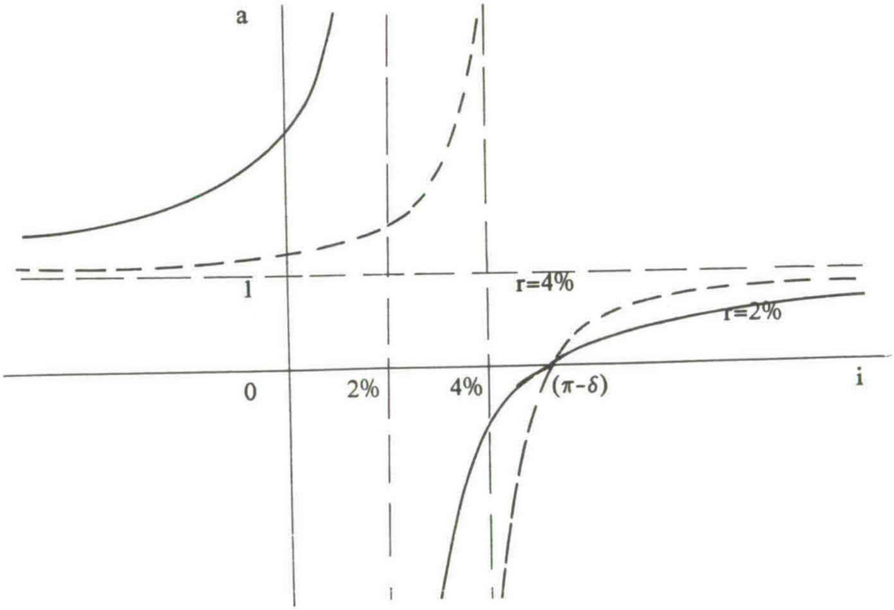
10 As we will see the optimum growth rate will always exceed the interest rate (i>r) if $(\pi-\delta)>r$. In order to ease our argument the explanation of the model is confined to this situation. The analysis is, however, also valid for the case with $(\pi-\delta)<r$. When discussing the optimum growth rate in section 3.6 we shall return to this case.

debtor. If moreover, the rate of interest exceeds the retained profits ($r>(\pi-\delta)$) the rate of indebtedness will rise above unity, leading to insolvency of the firm.

Equation (3.6) implies that, if $(\pi-\delta)>r$, a higher growth rate implies a higher debt ratio. At a given pay-out rate a higher indebtedness is thus the price that has to be paid for a higher growth rate in the long term. This relation between growth and debt is illustrated in *figure 3.2*. Notice that $\lim_{i\to\infty} a = 1$, which means that there exists no absolute financial limit on the growth rate.[11] Further, this figure brings out that $a\to-\infty$ if i approximates to r.

The profit rate has, of course, a negative effect on the debt ratio: if π is higher, less external finance is needed entailing a smaller debt ratio in the long run. The influence of the interest rate is ambiguous: if the firm is a net debtor a higher interest rate implies a higher steady state debt ratio; but, if the firm is a net creditor a higher interest rate raises income of the firm and thus leads to a *lower* debt ratio (in casu a larger credit position) in the long term. This is illustrated by the shift in the curve in *figure 3.2*.

Figure 3.2 Growth and debt



Explanation: this figure is based on the following numerical values: π=0.10; δ=0.05; and r=0.02 or 0.04 (in both cases $(\pi-\delta)>r$).

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11 This result shows that Williamson (1966) is wrong when arguing that there exists an absolute limit on the growth rate due to the requirement that the growth rate should not exceed the retention rate. If one allows for external debt, the requirement of positive net returns implies $i < \{(\pi-\delta)-ra\}/(1-a)$ which is not limiting at all if a is free.

3.5 GROWTH-RISK FRONTIER

This simple relation between the rate of indebtness and growth offers a good point of departure for the analysis of the long-term strategy of the firm. In the presence of equity rationing it is evident that there exists a positive association between the rate of indebtness and the financial risk of a firm.[12] As we do not pursue a fully-fledged assessment of the risk-posture of a firm, we shall adopt a simple device to introduce financial risk in the present model. As mentioned above, we concentrate on borrowers' risk; it is assumed that lenders are willing to supply funds unlimitedly at a given rate of interest. Further, we adopt the conventional proposition that risk is adequately measured by the second moment of the probability distribution (the variance).[13]

Now let the profit rate and interest rate be stochastic variables with known mean π and r and variances var(π) and var(r).[14] Given the budget constraint, shocks in these variables must be reflected either in distributed profits δ or in net investment i, or in the growth of debt Da. As it is well-established empirically that corporate dividends are sticky[15] we shall treat this variable as a constant in the short term. Further, we shall follow Kalecki (1937) and Wood (1975) and assume that firms hold on to a given

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12 The reasons for a positive relation between risk and the debt ratio are obvious: a large indebtedness implies that a large part of net earnings has to be spent on fixed debt service, so that the remaining flow of earnings becomes more sensitive to the volatility of the profit and the interest rate. Further, high debts may worsen the creditworthiness of the firm and thereby reduce its capacity to raise external funds. Finally, a high indebtedness enhances the risk of illiquidity and thus, in the event, the risk of bankruptcy as well.

13 A more sophisticated assessment of risk is given by Blatt (1983). He stresses that in an environment characterized by 'Knightian' uncertainty firms will in the first instance attempt to avoid a disaster, i.e. such as failure of the investment project that would lead to bankruptcy of the firm. As a corollary the downward risk, i.e. the area under the lower tail of the probability distribution, is much more important than the first two moments of the probability distribution. Blatt shows that a modified pay-back period may in such circumstances - in the short term - offer a better criterion for investment than the usual present value method. In the long term the length of this pay-back period should be derived subject to the probability distribution of costs and returns. Because of the complexity of this problem his analysis remains rather sketchy on this point. Therefore we shall follow a different approach with a simple conception of risk which allows for an explicit analysis.

14 The variance of the interest rate is determined by the simultaneous distribution of the nominal interest rate (r+p) and the rate of inflation p, thus

$$var(r)=var(r+p)+var(p)-2covar(r+p,r).$$

The variance of the profit rate depends on the probability distribution of the wage rate, technical change and the prices of input and output (P). If the technique of production and the rate of depreciation are constant, the variance of π is given by

$$var(\pi)=y^2var(P)+l^2var(W)-2yl\,covar(P,W).$$

This expression shows that the variance of π is not independent of the choice of technique. As a corollary, the optimum technique of production will be dependent on the probability distribution of P and W. Although a further analysis of this effect of uncertainty on the choice of technique is certainly interesting, it would, however, make the present analysis unnecessarily complex and distract us from our main concern. Therefore it will be neglected subsequently, so that var(π) can be treated as a given and independent parameter.

15 The classical contribution in this field of research is, of course, Lintner (1956).

target for the rate of indebtedness in the short term, thus Da=0. This is motivated by the observation that firms only periodically reconsider their financial strategies.[16] Consequently, if a and δ are constant there is only one variable left which must absorb all shocks, namely the growth rate[17]; thus according to equation (3.5)

$$\tilde{i} = \frac{1}{1-a} (\tilde{\pi} - \delta - \tilde{r}a) \tag{3.7}$$

where the tilde ($\tilde{}$) denotes the stochastic variables.[18] As the variance of net returns is thus fully transmitted into the variance of i, we may conceive var(i) as an indicator of the risk posture of the firm. From (3.7) we find

$$\text{var}(i) = \frac{1}{(1-a)^2} \{(\text{var}(\pi)+a^2.\text{var}(r)-2a.\text{covar}(\pi,r)\} \tag{3.8}$$

As the debt ratio depends on the growth rate (eq. 3.6) this result implies that there exists a unique relation between the growth rate and the risk posture of the firm, measured by its variability. This relation will be called the *growth-risk frontier*. As *figure 3.3* brings out this growth-risk frontier generally has a positive slope, reflecting the fact that the risk increases as the growth rate is pushed up.

This general result, however, is not valid when the growth rate approximates to the real interest rate. In this region risk is *negatively* associated with the growth rate. This is because the firm is then a creditor, which has invested its wealth for the larger part in financial assets rather than in capital goods (note that $a \rightarrow -\infty$ if $i \downarrow r$, see also *figure 3.3*). As a result of this one-sided composition of the portfolio the benefits of diversification decline and the risk rises again when i tends to r. This region with a negatively sloping growth-risk frontier is, however, not very interesting as it will prove to be non-eligible for any risk-averse firm.

It may be noted that the positive association between var(i) and i is also an

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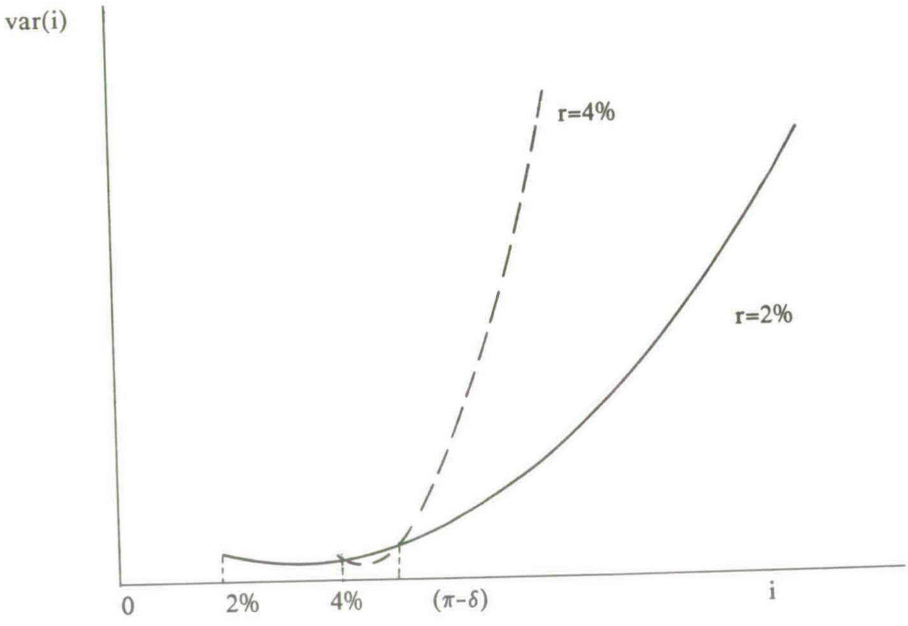
16 Apart from this theoretical motivation for a rigid debt ratio, this assumption also has a considerable technical advantage because a varying debt ratio would create complicated autoregressive processes which, unfortunately, cannot be reduced to a manageable ('Markov process') form on the analytical level.

17 Fazzari, Hubbard and Petersen (1988) argue that this is especially true for firms with high retention and low dividends. For mature, high dividend firms the relation between the variance in investment and net returns may be looser, as dividends may absorb part of the variability. Their empirical analysis indicates that variations in net return can explain a significant part of variations in the investment/capital ratio.

18 This result is similar to Lintner (1971) who also assumes a positive association between variations in profits and investment, but different from e.g. Baker (1978) who assumes that dividends absorb the shocks in the firm's earnings. For empirical support of the short-term relation between profits and investment on the microeconomic level of the firm see e.g. Eatwell (1971) and Fazzari, Hubbard and Petersen (1988).

essential element in the analysis of Lintner (1971). He does not, however, motivate this relation on the basis of theoretical considerations, but on the basis of an observed relation between these variables in practice: "Empirically there is evidence that larger retention undertaken to raise expected or average growth also leads to greater variability in the growth rates realized" (Lintner 1971, p.190).

Figure 3.3 Growth-risk frontier



Explanation: this figure is based on: π=0.10; δ=0.05; var(π)=0.006; var(r)=0.005; covar(π,r)=0.002; r=0.02 and 0.04.

3.6 OPTIMUM GROWTH

In managerial theories of growth it is assumed that managers aim at maximum growth or size of the firm in contrast with shareholders who desire maximum market value.[19] This is often motivated by the well-documented empirical observation that managerial rewards (including the non-pecuniary rewards such as status and perquisites) are related to the size of the firm rather than to its profitability (cf. Odagiri 1981). In the present analysis we shall follow this approach and, more precisely, assume that managers maximize the discounted sum of future sizes of the firm.[20] How the size must be measured is, however, not precisely clear. We shall consider two alternatives: the volume of production (Y) and the magnitude of capital stock (K). For the moment we concentrate on the latter. Then the optimization problem is:

$$\underset{i,l,\delta}{\text{Maximize}}\ PV(K) = \int_0^\infty K(t)\exp\{-\int_0^t \rho(\tau)d\tau\}dt \tag{3.9a}$$

$$\text{subject to } DK(t)=i(t).K(t);\ l(t) \geq 0;\ K(t) \geq 0$$

where ρ = rate of discount. The growth rate i, labour intensity l and pay-out δ are control variables; capital stock and the debt ratio are state variables. Since there are no adjustment costs with respect to capital and debt the firm can freely choose its initial capital stock subject to

$$K(0)= \frac{1}{1-a(0)} V_o \tag{3.10}$$

where V_0 is the given initial amount of net worth. As the initial value and the ultimate value of the state variables are free it can be shown that this problem is similar to the problem of choosing a once and for all constant growth rate i that maximizes[21]

19 Note that this representation of managerial preferences is different from that found in Jensen and Meckling (1976), who explain the divergence in interests between managers (inside owners) and shareholders (outside owners) by the desire of managers for non-pecuniary rewards (perquisites) at the expense of the current profits of the firm.

20 See Williamson (1966), Auberada (1979), Seoka (1985) for a similar assumption. As an alternative hypothesis it is sometimes assumed that managers aim at maximum (steady state) growth. As growth is generally easier as the initial size is smaller, this hypothesis, however, tends to give rise to unfortunate results. Solow (1971) and Auberada (1979) showed that the growth maximizing firm yields either the same growth rate as the size maximizing firm (if initial size is given), or gives rise to an infinitesimal small optimum initial size (if initial size is free).

21 For a proof see Appendix 3.A.

$$v = \frac{1}{V_o} \int_0^\infty K(0)\exp\{(i-\rho)t\}dt \tag{3.9b}$$

subject to $K(0) = V_o/(1-a)$; $K(0) \geq 0$
$a = \{i-(\pi-\delta)\}/(i-r)$

where v (= $PV(K)/V_o$) is the valuation ratio of managers, i.e. the ratio of the discounted size of capital stock and the given initial net worth V_o (>0). The last constraint restates the steady state budget constraint (eq. 3.6).

The important question to be answered now is what holds back managers in their desire to maximize the size of the firm ad infinitum. Given the budget constraint it is obvious that a higher growth rate implies either a higher debt ratio, or a lower pay-out to shareholders. A higher debt ratio is not attractive to managers as it raises the financial risk for the firm, and thus the risk for the management as well. But it is evident that the alternative of cutting the pay-out to shareholders may also affect the position of the management. If shareholders become discontent with the firm's policy, they have the ability to intervene and even to dismiss managers. Therefore, managers must take account of the interests of shareholders too.

This chapter will concentrate on the first constraint, i.e. the increasing financial risk as growth is higher. The other constraint, the necessity to keep shareholders satisfied is considered in the next chapter. For simplicity the representative firm in the present chapter is assumed to be a relatively small firm, which is owned by a small, steady group of shareholders (for example a family). The pay-out of profits is for the moment taken to be a fixed proportion of total profits.

Financial risk

For the incorporation of risk we follow - with slight modifications - the 'certainty equivalent' method of Lintner (1971). Starting from an exponential utility function $U(x)= -x^{\alpha}$ this author has shown that the certainty equivalent i^* of a stochastic, normally distributed, growth rate $\tilde{i}$ is given by

$$i^* = E(\tilde{i}) - \tfrac{1}{2}\alpha \, var(i) \tag{3.11}$$

Assuming a constant $E(\tilde{i})=i$ and a constant var(i) the present value PV of a variable x(t) is given by

$$PV(x(t)) = x(0).\exp\{(i - \tfrac{1}{2}\alpha \, var(i) - \eta)t\} \tag{3.12}$$

where η indicates the rate of time preference. This expression can be reduced by taking time preference η and risk premium $\tfrac{1}{2}\alpha var(i)$ together in the risk adjusted discount rate ρ, thus

$$\rho = \eta + \tfrac{1}{2}\alpha \operatorname{var}(i) \tag{3.13}$$

Now our model is complete and can be solved. After substitution for K(0) and assuming convergence of the integral, the optimization problem (3.9b) can be reduced to

$$\underset{i,l}{\text{Maximize}}\ v = \frac{1}{\rho - i}\ \frac{i-r}{i-(\pi-\delta)} \tag{3.14}$$

subject to $\rho = \eta + \tfrac{1}{2}\alpha \operatorname{var}(i)$ (eq. 3.13)
$\pi = (1-\tau_\pi)(y-Wl-\psi)$ (eq. 3.2)

where var(i) is a function of the debt ratio (eq. 3.8) and y is given by the production function $y(l)$ (eq. 3.4). The rate of investment i and employment l are control variables; the wage rate W, the pay-out rate δ and the interest rate r are given.

Note that this procedure is only valid if $i<\rho$ for all i. If there exists some $i \geq \rho$ the optimization problem (3.9b) becomes subject to the *growth-stock paradox*; that is, if the integral does not converge v becomes infinite for a range of instrument settings, so that no unique optimum can be determined.[22] As in the present model the risk premium var(i), and thus ρ, rises faster than i when $i \to \infty$, so that

$$\lim_{i\to\infty} (i-\rho) = -\infty$$

it can be assessed that this model may produce a finite optimum without the growth stock paradox.[23]

The basic considerations regarding the optimum growth rate can be discussed with reference to the scheme of the causal relationships below. A higher growth rate has a positive effect on v through its effect on the future scale of the firm, and on the initial scale as well (as a result of the higher debt ratio). On the other hand, however, a greater indebtedness also affects the risk posture of the firm, thus raising the discount rate. For low growth rates the positive effects outweigh this latter negative

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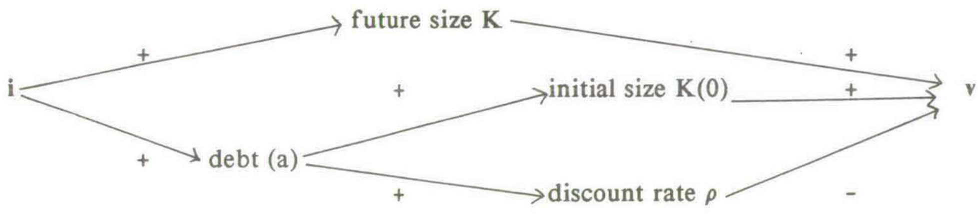
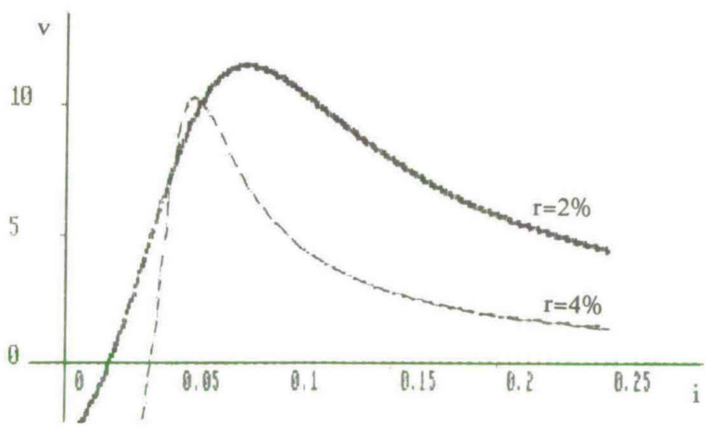
22 As Lintner (1971) has shown, this paradox hinges on the assumption of a constant rate of discount over time. Lintner rightly argues that this is unrealistic because in reality uncertainty increases as the future becomes more distant. As an alternative he shows that if i behaves as a random walk, so that its variance increases linearly with time, the growth-stock paradox is excluded for any i. Although his solution is elegant, it is too complex to handle in more elaborate models. Moreover, Lintner seems unaware that if ρ increases with time, the constant growth path no longer corresponds to the dynamic optimum. Therefore a more general formulation of the optimization problem is necessary. Because of these complexities we shall follow common practice, and assume that the parameters of the ρ-i relation are such that $\rho - i > 0$ for all $\{i, l\}$.

23 This is obtained from equations (3.6, 3.8 and 3.13).



effect, but beyond some point the negative effect of i on the risk will become dominant. This is corroborated by the v-i frontier in *figure 3.4* which is based on a numerical simulation of equation (3.14). Just as the growth-valuation frontier in conventional corporate models, so this i-v frontier is concave for any i>r.[24] However, in our model this concavity is caused by increasing risk whereas in corporate theories it is usually explained by declining profitability as i increases.

Figure 3.4 Growth and valuation



Explanation: this figure is based on the numerical values given in figure 3.3 and α=10; η=0.15 and r=0.02 and 0.04.

24 This i-v frontier should not be confused with the growth-valuation frontier in most other managerial theories of growth, where valuation refers to the valuation of shareholders.

Optimum growth rate

The first order conditions for an optimum (written in a convenient fashion) are

$$\frac{\partial v}{\partial l} = \frac{\partial v}{\partial \pi} \cdot \frac{\partial \pi}{\partial l} = 0 \tag{3.15}$$

$$\frac{\partial v}{\partial i} = v\{\frac{1}{i-r} - \frac{1}{\pi-i}(\frac{\partial \pi}{\partial i} - 1)\} = 0 \tag{3.16}$$

The first condition yields the familiar marginal productivity rule for instant profit maximization $\partial\pi/\partial l=0$, which implies

$$y' - W = 0 \tag{3.17}$$

The second condition can after some manipulation be reduced to the following expression for the optimum growth rate i^* [25]

$$i^* = r + \{(\pi-\delta)-r\}\{\frac{var(r) + 2(\eta-r)/\alpha}{var(\pi) + var(r)-2\ covar(\pi,r)}\}^{1/2} \tag{3.18}$$

This result implies that $i^*>r$ whenever $(\pi-\delta)>r$, and $i^*<r$ otherwise. Further, the characteristics of this optimum can be found from the partial derivatives of i^*

$\partial i^*/\partial\pi > 0$		
$\partial i^*/\partial r > 0$		if $a > -(1-a)^2\{(\pi-\delta)-r\}$
$\partial i^*/\partial\alpha < 0$;	$\partial i^*/\partial\eta > 0$	if $a > 0$ and $(\pi-\delta)-r > 0$
$\partial i^*/\partial covar(\pi,r) > 0$;	$\partial i^*/\partial var(\pi) < 0$	if $a > 0$ and $(\pi-\delta)-r > 0$
$\partial i^*/\partial var(r) < 0$		if $a > 0$ and $(\pi-\delta)-r > 0$

Most results conform intuition. The optimum growth rate is positively associated with the rate of retained profits and the time preference.[26] The sign of the interest rate is

25 Equation 3.16 actually gives two solutions for i, but the lower solution corresponds to a minimum. It may be noted that the solution (3.18) is also valid for the case with $(\pi-\delta)<r$. In this case the growth-risk frontier is characterized by $var(i)\rightarrow\infty$ if $i\rightarrow-\infty$ and $var(i)\rightarrow var(r)$ if $i\rightarrow r$. As risk is still a concave function of i, the model will also in this case generally yield an interior solution for i. In this case the optimum for i will be less than the real interest rate, $i^*<r$.

26 Note that equation (3.18) implies that the optimum debt ratio is independent of π and δ,

$$a^* = 1 - \{var(\pi) + var(r) - 2covar(\pi,r)\}^{1/2}/\{var(r) + 2(\eta-r)/\alpha\}$$

ambiguous. When the firm has a net creditor position[27] and the rate of retained profits is close to the interest rate a rise in the interest rate may lead to a *higher* growth rate. Elsewhere the effect of r is negative as usual. As profits become more volatile (var(π) higher) the firm will choose a lower growth rate. The same happens when the volatility of the interest rate goes up, except when the firm is a net creditor; in the latter case the firm will reduce its risk by reducing its net creditor position and thus by choosing a *higher* growth rate. An increase in the covariance between π and r, which lowers the risk, always leads to an increase in the growth rate. Finally, as managers have a stronger time preference (η greater) or a smaller risk aversion (α lower) they will choose a higher growth rate.

All these effects have been discussed for the normal case in which net retained profits exceed the interest rate. If $(\pi-\delta)<r$ a higher growth rate is, as we have seen, associated with a lower steady state debt ratio. It is obvious that many of the above effects may then change direction (see eq. 3.18).

Production maximizer

If managers measure the firm's size by its production rather than by its capital stock the results will be slightly different. The production maximizing firm (YMF) maximizes the size of discounted future production in relation to initial net worth, thus provided that the growth stock paradox does not occur:

$$\underset{i,l}{\text{Maximize}}\ v_{YMF} = \frac{y}{\rho - i}\,\frac{i-r}{i-(\pi-\delta)} \tag{3.14c}$$

The first order condition with respect to i is identical to the condition for the capital maximizing firm (KMF) (eq. 3.16 above), and thus yields the same result for the optimum growth rate as above (eq. 3.18). However, the condition with respect to the choice of technique is different and becomes

$$\frac{v}{y}y' + \frac{\partial v}{\partial \pi}\frac{\partial \pi}{\partial l} = 0$$

Since $(v/y)y'>0$ and $\partial v/\partial\pi>0$ this condition can be seen to require $\partial\pi/\partial l<0$; this means that the production maximizing firm always chooses a *more labour-intensive technique* than the KMF. As a corollary the marginal productivity of labour will be less than the wage rate ($y'-W<0$), and the optimum profit rate lower than the maximum profit rate.

It varies positively with time preference η and the covariance between π and r and negatively with the interest rate and the variance of π.

27 The firm is a net creditor (a>0) if $r > \eta - \frac{1}{2}\alpha\{var(\pi)-2covar(\pi,r)\}$.

Moreover, since the growth rate varies with the profit rate (eq. 3.18) it can be concluded that the YMF also chooses a lower growth rate than the KMF. Thus in summary:

$$l_{YMF} > l_{KMF}; \quad \pi_{YMF} < \pi_{KMF}; \quad i_{YMF} < i_{KMF}$$

Aside, it may be noticed that the choice of technique has become interdependent with the growth strategy of the production maximizing firm; all factors affecting the growth-risk trade-off thus influence the choice of technique as well.

In order to get an impression of the differences between the YMF and the KMF, *table 3.1* presents some numerical results for the optimum values i*, l* and π* for different wage rates and interest rates, assuming a Cobb-Douglas production function $y=l^{\beta}$. These results confirm our theoretical findings for the KMF and the YMF. Although the differences are quite small, it can be seen that the YMF chooses a higher n and a lower π, and consequentially a lower i than the KMF. These results also show that the technique of production of the YMF is dependent on the interest rate, while it is independent for the KMF. Further, these numerical results indicate that the labour intensity, the profit rate and the growth rate fall as the wage rate rises. As in our theoretical analysis above, the impact of the interest rate on the growth rate does indeed depend on the financial position of the firm. If the firm is a net debtor a higher interest rate reduces the growth rate, but when it has a net creditor position and the profit rate is low relative to the interest rate it may induce a higher growth rate.

Table 3.1 Numerical results for the KMF and the YMF (between brackets) for r=4% and r=10%

W	r(%)	π*(%) KMF (YMF)	l* KMF (YMF)	i*(%) KMF (YMF)
0.8	4	22.0 (22.0)	0.64 (0.68)	14.9 (14.9)
	10	22.0 (21.8)	0.64 (0.72)	10.8 (10.8)
1.2	4	8.5 (8.2)	0.17 (0.23)	4.4 (4.1)
	10	8.5 (8.1)	0.17 (0.24)	5.2 (5.0)
1.6	4	4.4 (4.2)	0.06 (0.09)	1.2 (1.0)
	10	4.4 (4.2)	0.06 (0.09)	3.4 (3.3)
2.0	4	2.6 (2.5)	0.03 (0.04)	-0.2 (-0.3)
	10	2.6 (2.5)	0.03 (0.04)	2.6 (2.6)

Explanation: β=0.7 and all other numerical values as in figure 3.3. At these parameters the debt ratio is 0.36 (if r=4%) and -0.18 (if r=10%).

3.7 MONITORING AND THE PAY-OUT OF PROFITS

From the point of view of the shareholders the firm's strategy is optimal if the present value of paid out profits is maximized in relation to initial net worth, thus

$$\text{Maximize} \quad q = \frac{1}{V_o}\int_0^{\infty} \delta K(0)\exp\{(i-\rho_s)t\}dt \tag{3.9c}$$

$$\text{subject to} \quad K(0) = V_o/(1-a)\ ;\ K(0) \geq 0$$

where ρ_s stands for the risk adjusted discount rate of shareholders ($\rho_s = \eta_s + \frac{1}{2}\alpha_s.\text{var}(i)$). Since for any given δ the pay-out of profits is proportional to capital stock, this q ratio is maximized if the present value of future capital stocks is maximized. If shareholders have the same time preference and risk aversion as managers ($\eta_s=\eta$; $\alpha_s=\alpha$) this optimization problem therefore yields the same technique of production and growth rate as for the KMF (eqs. 3.17, 3.18). However, if shareholders are more risk averse or have a smaller time preference they would prefer a lower debt ratio and thus a lower growth rate than managers.

There is, however, a more fundamental conflict of interests between managers and shareholders; this concerns the determination of the pay-out of profits δ. It can be established that the present model yields no interior solution for the optimum δ for shareholders. Solving equation (3.9c) for a constant growth rate and rewriting q as a function of δ and a, we find

$$q = \frac{\delta}{\delta + (1-a)\rho_s - (\pi - ar)} \tag{3.19}$$

Since ρ_s is a function of a and independent of ρ_s, maximization of this function with respect to δ gives either an infinite or an undetermined pay-out rate. Whenever the discount rate exceeds the rate of return on net worth ($\rho_s>(\pi-ar)/(1-a)$) the q ratio tends to a maximum ($q_{max}=1$) if $\delta\rightarrow\infty$, i.e. if shareholders completely withdraw their investments from the firm. On the other hand if $\rho_s<(\pi-ar)/(1-a)$ the optimization problem is subject to the growth-stock paradox[28], so that no unique optimum can be established.

Figure 3.5 shows the valuation ratios v and q as a function of the pay-out rate δ for the case with $\rho_s>(\pi-ar)/(1-a)$. The conflict of interests between managers and shareholders is evident from this figure. It is obvious that managers prefer δ to be as

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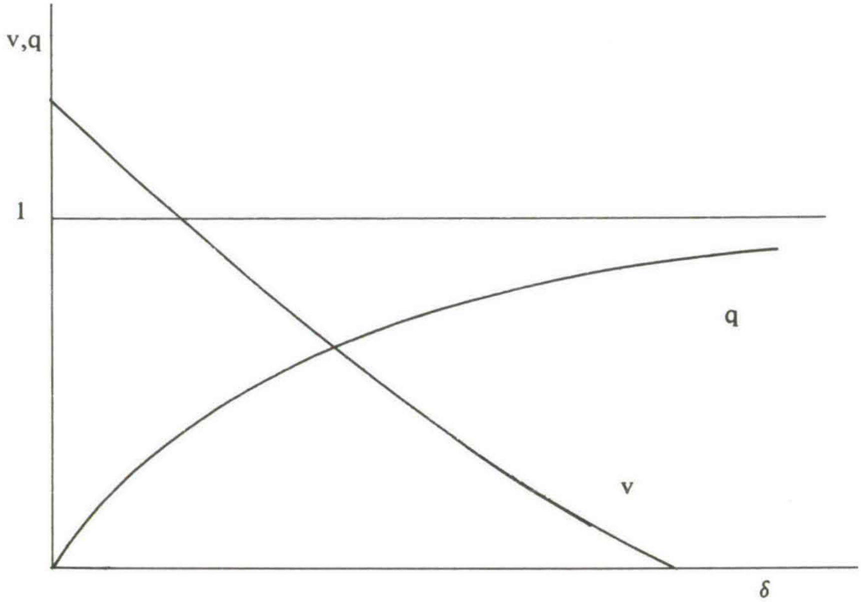
28 As the budget constraint (eq. 3.6) implies $i=(\pi-\delta-ar)/(1-a)$ it can be seen that when $\rho_s<(\pi-ar)/(1-a)$ there must exist some $i>\rho_s$ for $\delta>0$ so that $\rho=\infty$ (see eq. 3.9c).

Figure 3.5 Valuation ratios of managers and shareholders



low as possible, while shareholders prefer the highest possible pay-out rate. What pay-out of profits is actually realized depends on the discretionary power of managers vis-à-vis shareholders. This depends on a variety of factors, such as the ownership share of managers (Jensen and Meckling, 1976), competition on the labour market for managers (Fama, 1980), the legal and organizational structure of the firm, the role of credit institutions, information costs, etc.

By way of tentative analysis we shall consider a simple, and obvious, relationship between the monitoring effort of shareholders and the discretionary power of managers. Under the plausible assumption that monitoring efforts are subject to decreasing marginal returns it can be shown that the optimum is reached where the marginal returns of monitoring in terms of a higher q are equal to the marginal cost of monitoring. For example, assume the following relation between the cost of monitoring as a fraction of capital stock (κ) and the deviation of the managerial valuation ratio v from its maximum

$$\kappa = \kappa(v_{max}/v) \qquad \kappa'>0;\ \kappa''>0;\ \kappa(1)=0 \tag{3.20}$$

This function states that in the absence of monitoring (κ=0) the managers will choose the strategy that maximizes v, hence $(v_{max}/v)=1$. In order to force the management to choose a strategy yielding a lower v ($v_{max}/v > 1$) a positive monitoring effort by shareholders will be necessary. Since v is maximal if $\delta=0$ the ratio between v and v_{max} can, after substitution, for i be written as a function of δ

$$\frac{v_{max}}{v} = \frac{x + \delta}{x} \qquad \text{where } x = (1-a)\rho-(\pi-ar) \tag{3.21}$$

From the point of view of shareholders the cost of monitoring should be subtracted from the pay-out of profits. Hence the q ratio (eq. 3.19) becomes

$$q = \frac{\delta - \kappa}{x_s + \delta} \qquad \text{where } x_s = (1-a)\rho_s-(\pi-ar) \tag{3.22}$$

The first order condition for the optimum pay-out rate now yields[29]

$$\frac{\partial q}{\partial \delta} = \frac{(x_s+\kappa) - (x_s+\delta)\kappa'/x}{(x_s+\delta)^2} = 0 \tag{3.23}$$

This result shows that there will always exist an interior solution if $\kappa',\kappa''>0$ and $\partial q/\partial\delta>0$ at $\delta=0$. This latter condition states that the marginal cost of monitoring at $\delta=0$ should not exceed its marginal return in terms of pay-out ($\kappa'(0)<x$). Otherwise, there would be no monitoring at all (hence $\delta=0$). Equation (3.23) implies that the optimum pay-out is higher as the marginal cost of monitoring is less, and the potential benefits are greater, i.e. if the difference between the discount rate and the rate of return on net worth is larger (x_s or x higher).

In this deterministic approach shareholders are assumed to have full insight into the strategy of managers and the costs and benefits of monitoring. The pay-out is then determined simultaneously with the growth strategy of the firm. Such an optimization procedure is, however, difficult to imagine in practice, where shareholders have only limited knowledge and monitoring is a complex process with many different actors. Moreover, shareholders are not a homogeneous group, but have different amounts of shares, different time preference and risk aversion, and different ideas about the costs and returns of monitoring activities. All these factors make the organization of

29 If shareholders have the same discount rate as managers there is, as we have seen, no divergence of interests with respect to the optimum debt ratio (hence $\partial q/\partial a=0$ whenever $\partial v/\partial a=0$). As in this case the solution for a is independent of δ (see footnote 26 above), the optimization problem can be solved in two separate steps, as we have done in this chapter. If $\rho_s \neq \rho$ the monitoring effort will, however, also be directed at the debt ratio of the firm. In that case the optimum debt ratio is no longer independent of the payout rate, and needs to be determined simultaneously with the condition for the optimum payout rate. For brevity, however, this is neglected in the present tentative analysis.

monitoring more complex. This may provide an ex post rationale for the assumption of a given pay-out rate used in the foregoing analysis. If monitoring is difficult to organize it seems reasonable to assume that shareholders agree on a fixed pay-out rate which is maintained as long as their confidence in the management is not shaken.

3.8 CONCLUSION

In this chapter starting from a basic model of a permanently growing, equity rationed firm, a unique relation has been established between the growth and the financial risk of the firm. On the basis of this growth-risk frontier we have established the optimum growth rate as a function of retained profits, the interest rate and the risk ensuing from the variability of profits and the interest rate, given the time preference and risk aversion of the managers.

Unlike most conventional models of corporate growth the present model does not require a negatively sloped growth-profitability frontier. In stressing increasing risk rather than the declining rate of return our analysis builds on Kalecki's *principle of increasing risk* rather than on Keynes' proposition of a falling marginal efficiency of investment. This does not mean, however, that the growth-profitability relation is inconsistent with the present approach. On the contrary, this frontier and the growth-risk frontier are natural counterparts in the determination of the growth of firms. Therefore in the next chapter the consequences of incorporating the growth-profitability frontier in the present model will be investigated.

The model considered in this chapter was also restrictive in two other important respects: in the first place it neglected the possibility of raising funds by floating shares; secondly, the analysis was limited to the steady state, and it neglected the adjustment process. The next chapter will extend the analysis to these points as well.

APPENDIX 3.A DYNAMIC OPTIMUM

This appendix shows that the time path obtained under the restriction of a once and for all constant growth rate corresponds to a true dynamic optimum if the initial value of a and K are free and if prices and preferences are constant over time. A general formulation of the optimization problem is

$$\text{Max } v = \int_0^\infty K(t).\exp\{-\int_0^\infty \rho(\tau)d\tau\}dt \tag{A.1}$$

Since $K(t) = V(t)/(1-a(t))$
and $DV(t) = V(t)\{(\pi-\delta)-a(t)r\}/(1-a(t))\}$

(where D is the differential operator, and π, δ and r are constant over time) equation A.1 is equivalent to

$$\text{Max } v = \int_0^\infty \frac{V_o}{1-a(t)} z(t)\, dt \tag{A.2}$$

subject to

$$z(0)=1 \quad \text{and} \quad Dz(t) = \{\frac{(\pi-\delta)-a(t)r}{1-a(t)} - \rho(t)\}z(t) \tag{A.3}$$

where z is a state variable and a the control variable. Suppressing the time subscripts the Hamiltonian system is

$$H = \frac{V_o}{1-a} z + \lambda\{\frac{(\pi-\delta)-ra}{1-a} - \rho\}Dz \qquad \text{subject to } z \geq 0;\ z(0)=1;\ a<1 \tag{A.4}$$

The first order conditions are

$$\frac{dH}{da} = \frac{V_o}{(1-a)^2} z + \lambda\{\frac{(\pi-\delta)-r}{1-a} - \rho_a\}z = 0$$

$$\frac{dH}{dz} = \frac{V_o}{(1-a)} + \lambda\{\frac{(\pi-\delta)-r}{1-a} - \rho\} = -D\lambda$$

where ρ_a represents the first derivative of ρ with respect to a. These conditions yield the following result for the debt ratio:

$$a = \{(1-a)\rho_a - \rho + r\} \frac{(\pi-\delta)-r-(1-a)^2\rho_a}{2(1-a)\rho_a-(1-a)^2\rho_{aa}} \tag{A.5}$$

where $\rho_{aa} = \partial^2\rho/(\partial a)^2$.

Now consider optimization of v subject to the steady state assumption of a constant growth rate, or in terms of the present model, of a constant debt ratio (Da=0). From equation A.2 we then obtain:

$$v = \frac{V_o}{1-a}\left\{\frac{(\pi-\delta)-ra}{1-a} - \rho\right\}^{-1}$$

The first-order condition dv/da=0 is satisfied if

$$(1-a)\rho_a - \rho + r = 0$$

Comparing this result with equation (A.5) proves that this solution derived from the steady state proposition indeed satisfies the condition for a true dynamic optimum. Q.E.D.

CHAPTER 4

GROWTH OF THE FIRM: SOME EXTENSIONS

4.1 INTRODUCTION

The basic model described in the foregoing chapter will now be elaborated with respect to three important points. First, we shall relax the assumption of full equity rationing and introduce the possibility of issuing new shares (section 4.2). Secondly, we analyse the consequences of positive costs of growth for corporate strategy and the conflict of interests between managers and shareholders (section 4.3). Finally, in section 4.4 we shall drop the assumption of a free initial capital stock and examine the adjustment process starting from a given initial size of the firm.

4.2 MARKET VALUATION AND CORPORATE STRATEGY

So far we have concentrated on a relatively small firm which has no access to the equity market at all. Introduction of the equity market adds two important dimensions to our analysis. First, the possibility of issuing new equity introduces an additional source of finance to the firm besides internal saving and external debt. Secondly, the market valuation of shares becomes an important signal to managers as an indicator of the risk of intervention by the shareholders or take-over by new owners.

This second aspect is probably more important than the role of the equity market as an additional source of finance. In practice funds raised by floating shares make up only a very minor part of total funds. In the post-war period non-financial firms in the United States raised some 60 to 70 per cent of their funds internally, 30 to 40 per cent by external debt and only 1 to 6 per cent by new equity (Taggart 1986, p. 19). Looking at the *net* financing of firms, i.e. gross financing less accumulation of financial assets, the figures are even more revealing. In a recent survey Mayer (1988) reports that the average contribution of shares in the period 1970-1985 ranges from -4 per cent in the UK to 5 per cent in France. Retentions are by far the most important source of finance ranging from some 60 per cent in France and Japan to more than 100 per cent in the UK (see *table 4.1*).

These figures suggest that selling new shares is not a normal way of funding investment. Mayer (1988, p.1189) accordingly concludes that "the issuance of stocks is much more related to the problem of power in the firm and, in this respect, is restricted to very special moments in the life of the firm."

Table 4.1 Net financing of private physical investment 1970-1985 (percentage of total finance)

	retentions	debt	shares
France	62	32	5
Germany	73	26	1
Japan	65	31	4
UK	107	-3	-4
USA	90	13	-3

Source: Mayer (1988, p. 1174)

Nevertheless the equity market plays an important role in corporate strategy, not so much as a source of finance, but rather because of the impact of market valuation on the discretionary power of managers. If the managerial strategy presses the market valuation too far down this may provoke a reaction by shareholders to reduce the power of the present management, or even to dismiss them. In addition, a low valuation makes the firm more susceptible to take-overs, as outsiders can make a capital gain by taking over control of the firm.

In this section we shall investigate the influence of market valuation on the strategy of a large corporate firm which has access to a well-developed equity market. Nevertheless the equity market is imperfect in the sense that there is only a limited demand for the firm's shares.[1] This imperfection manifests itself in a falling demand curve for the firm's shares. This is in accordance with the empirical observation that share prices fall as more new equity is issued,[2] which is usually explained by the greater reliance on more pessimistic and risk averse agents when the floatation of equity is increased (cf. Nickell 1978, p. 184).[3]

Market valuation

The falling demand curve for shares can be represented by a rising rate of discount as the amount of equity issued increases. For simplicity, it is assumed that shareholders are fully aware of the mutual relationship between new equity raised and dividends paid out. Neglecting liquidity constraints and distorting taxation, the net amount of dividends and new equity raised can then be considered as a homogeneous

[1] As mentioned in chapter 3 (note 7) the proposition of a perfect equity market is inconsistent with the managerial approach which stresses the relevance of the ownership structure of the firm.

[2] For recent evidence on the falling demand curve of shares see e.g. Shleifer (1986)

[3] Another explanation can be derived from agency theory. As monitoring and bonding costs increase when more new shares are issued, the market valuation, which is based on returns net of these costs, will fall.

variable.[4] This difference between the gross pay-out of profits and new equity raised will in the following discussion be represented by the pay-out rate δ, which is now conceived as a *net* rate.

The functional relationship between the market rate of discount ρ_s and the *net* pay-out rate δ may - modifying equation (3.13) of the foregoing chapter - be written as[5]

$$\rho_s = \eta_s + \tfrac{1}{2}\alpha_s.\mathrm{var}(i) + \sigma_s(\delta) \qquad \sigma_s'<0;\ \sigma_s''>0 \tag{4.1}$$

where the suffix s refers to shareholders. As in our basic model var(i) is again determined by the debt ratio and the probability distribution of π and r (eq. 3.8). Given this discount rate and a constant growth rate i the market valuation ratio q is

$$q = \frac{1}{V_o} \int_0^\infty \delta K(0) \exp\{(i-\rho_s)t\}\, dt \tag{4.2}$$

where V_o is the initial net worth. The convexity of the relation between the rate of discount and the net pay-out rate ensures that an interior solution can exist for the maximization of q, that is, the optimum strategy from the point of view of shareholders (for a proof, see *Appendix 4.A*).

Managerial strategy

As we have mentioned the valuation of shares on the equity market imposes a significant restraint on the discretionary power of managers. The precise modelling of this restraint is not generally agreed on, but it is widely accepted that it arises from fear of dismissal by dissatisfied shareholders or take-over by new owners when the

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[4] This can be shown as follows. Defining J=number of shares, P_e=price of shares, d=dividend ratio and j=DJ/J, then the market valuation of all outstanding shares is given by the present value of dividends on these shares:

$$P_eJ = {}_0\int^\infty dK(0).\exp\{(i-j-\rho)t\}dt$$

Under the usual assumptions this can be solved into

$$PeJ = dK(0)/(\rho+j-i)$$

Since we have defined pay-out of profits net of equity raised ($\delta K = dK - jP_eJ$), we find for the total value of shares $P_eJ = \delta K(0)/(\rho-i)$. This latter result shows that the market valuation P_eJ depends on net payout only, and is thus independent of the division of net returns $\delta K(0)$ in dividends dK(0) and new equity raised jP_eJ.

[5] For simplicity we neglect the relation between the risk premium and the probability distribution of the market rate of return. According to the CAPM model only the non-diversifiable part of the variance of i is relevant to the risk premium. Incorporation of this relation would, however, not essentially change the subsequent analysis. For an appropriate account of this relation in the context of a similar model, see Lintner (1971).

market value of shares is pressed too far down. Some authors postulate an absolute minimum for the market value (cf. Marris 1971, Uzawa 1969, Solow 1971, Slater 1980), while others define a minimum for the ratio between the market value and the potential maximum value (Williamson 1971).

A more sophisticated approach is given by Odagiri (1981), who shows that in an uncertain environment the probability of take-over should be related to its expected costs and benefits. As the costs will be fairly constant over time and the benefits vary directly with the divergence between the actual market value and the potential value, he argues that this divergence is the most important determinant of the risk of take-over.

In the following analysis we shall introduce this risk for managers by a hazard rate σ reflecting the chance of being dismissed. Following Odagiri this hazard rate is assumed to depend on the market valuation q and the potential maximum valuation q_{max}. As the utility of managers after they have been dismissed is zero, the 'effective' future size of the firm for the present management should be discounted by this hazard rate in addition to time preference and risk premium, thus

$$\rho = \eta + \tfrac{1}{2}\alpha.\mathrm{var}(i) + \sigma(q_{max},q) \qquad \sigma_q' < 0;\ \sigma(q=0) = \infty;\ \sigma(q=q_{max}) = 0 \tag{4.3}$$

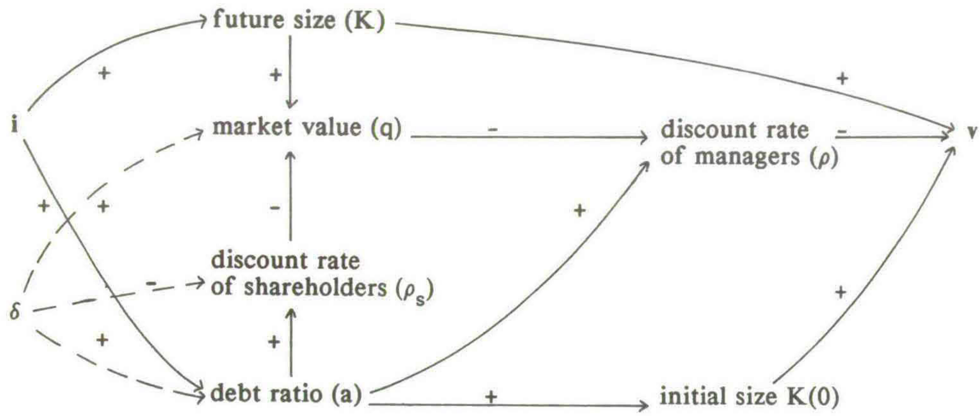
where q_{max} follows from the maximization of equation (4.2) above.

The discount rate of managers now incorporates two kinds of risk: the financial risk (var(i)) which is related to the firm as a whole, and the managerial risk (σ) which is specifically attached to the position of the managers. The σ function reflects the ownership structure of the firm. If shareholders have full power over the firm σ_q' would be infinite at $q=q_{max}$; that is, all managers will immediately be dismissed at the slightest discrepancy between q and its maximum. If shareholders have no influence at all, σ_q' would of course be zero for all $\{q_{max},q\}$.[6]

Given this relation for ρ the optimization problem for the managers is to select a constant growth rate i, net pay-out rate δ and technique of production n that maximizes

$$v = \frac{1}{V_o} \int_0^\infty K(0) \exp\{(i-\rho)t\}\, dt \tag{4.4}$$

[6] This representation of the conflict between managers and shareholders focusses on the behaviour of managers, in contrast with the tentative 'monitoring cost' approach of section 3.7 in the foregoing chapter, which concentrated on the costs and benefits of shareholders.



The basic considerations with respect to the choice of i and δ can be explained with reference to the scheme above:[7] Starting from a low growth rate close to r (where a = $-\infty$) it can be assessed that an increase in i initially has a beneficial influence on v because of its positive effect on the initial and future scale of the firm. Also the market valuation q rises at first, thereby diminishing the managerial risk (σ) and pushing v up even more.[8] However, as i and thus the debt ratio are raised further, the negative effects of the increasing financial risk on the discount rates ρ and ρ_s become stronger, and will sooner or later outweigh the positive effects on the firm's size. Thereafter, further raising of i goes together with a fall in v. In the limit, if $i \rightarrow \infty$ and $a \rightarrow 1$ the discount rates of shareholders and managers tend to infinity, so that v and q fall to zero:

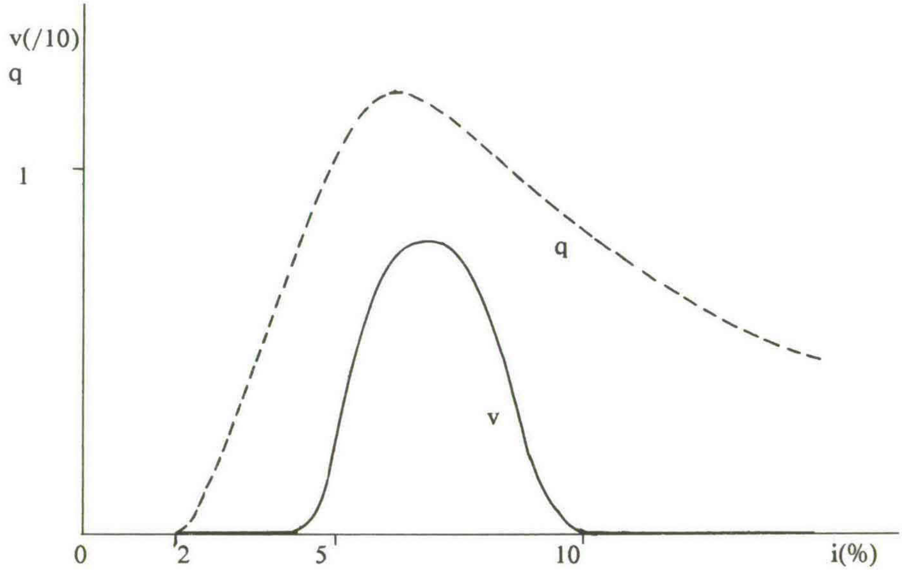
$$\lim_{i \rightarrow \infty} v = 0; \quad \lim_{i \rightarrow \infty} q = 0$$

The resulting growth-valuation frontiers for v and q are shown in *Figure 4.1.*

[7] For expository reasons the explanation again concentrates on the case with $(\pi-\delta)>r$ and $i>r$. As in the foregoing chapter the results are, however, also valid for $(\pi-\delta)<r$ and $i<r$.

[8] If $(\pi-\delta)>r$ the relevant region for i starts at $i=r$. At $i\downarrow r$ the debt ratio tends to $-\infty$ so that the variance becomes equal to var(r). Provided that covar(π,r)<var(r) the rise in i also contributes to a higher q and v because of the declining financial risk var(i) as i the debt ratio is raised .

Figure 4.1 Growth and valuation



Explanation: This figure is based on a numerical example with $\sigma_s = \nu.\exp(-\omega\delta)$; $\sigma = \theta[\exp\{\varphi(q(max)-q)/q\} - 1]$; δ=0.05; π=0.10; r=0.02; η=0.025; η_s=0.02; κ=0.01; φ=20; θ=15; ν=0.1; $\alpha=\alpha_s$=10; var(π)=0.006; var(r)=0.005; covar(π,r)=0.002. The σ_s and σ functions will be discussed below with reference to table 4.3.

With regard to the net pay-out rate we find the q ratio starts at zero at δ=0. As the risk of dismissal is then infinitely large, the valuation ratio of managers (v) starts at zero as well, thus

$$q_{|\delta=0} = 0\,; \quad v_{|\delta=0} = 0$$

As δ is raised above zero the q ratio starts to rise, and thanks to the falling risk of dismissal v rises as well. Beyond some point, however, the negative effects of δ on the debt ratio, and thus on var(i) become dominating, so that q and v begin to fall again. Eventually, when δ approximates $\delta_o = (\pi - r)$ where the debt ratio becomes equal to unity, the financial risk (var(i)) becomes infinitely large so that q and v fall to zero again (see *figure 4.2*), thus

$$\lim_{\delta\to\delta_o} q = 0; \quad \lim_{\delta\to\delta_o} v = 0 \qquad\qquad \delta_o = (\pi - r)$$

The convexity of these functions ensures that an interior solution exists to the maximization of v. Provided that the growth-stock paradox does not occur and that a finite maximum exists for q_{max}, the first-order conditions for v are

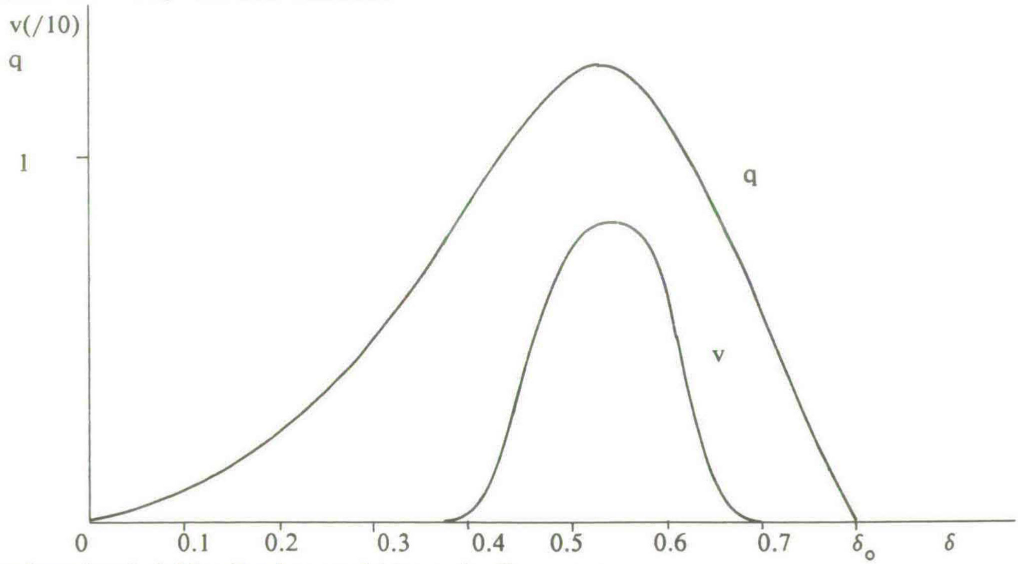
$$\frac{\partial v}{\partial l} = \frac{\partial v}{\partial \pi}\frac{\partial \pi}{\partial l} = 0 \tag{4.5}$$

$$\frac{\partial v}{\partial a} = v^2\{\rho - r - (1-a)\frac{\partial \rho}{\partial a}\} = 0 \tag{4.6}$$

$$\frac{\partial v}{\partial \delta} = -v^2\{(1-a)\frac{\partial \rho}{\partial \delta} + 1\} = 0 \tag{4.7}$$

For convenience the conditions have now been written with respect to l, δ and a. The growth rate is, of course, implied in the solution of these variables. The first condition determines the optimum technique of production (where y'=W). Both the other conditions determine the simultaneous solution for a and δ. As $v|_{q=0} = \infty$ and $v|_{a=1} = \infty$, the conditions (4.6) and (4.7) ensure that the managers always select a policy with positive net pay out ($\delta>0$) and a debt ratio less than unity ($a<1$), and thus a strategy with finite growth rate (i). Note that if the demand curve for the firm's shares does not fall (hence $\partial\rho/\partial\delta = 0$) the last condition (4.7) cannot be satisfied for any $v>0$. The assumption of a limited supply of new equity is thus a necessary condition for an interior solution of the firm's strategy to exist in this model.

Figure 4.2 Pay-out and valuation



Explanation: i=0.06; all other variables as in figure 4.1

Conflicting strategies
The differences between managerial strategy and the strategy desired by shareholders can be established by evaluating the first-order conditions for v in the optimum for q. If the conditions for v prove to be satisfied in the shareholders' optimum $\{l,a,\delta|q=q_{max}\}$, this will of course represent an optimum for the managers too. If the conditions for v are not satisfied at $q=q_{max}$, the managers will select a different strategy. After some manipulation it can be found for the first derivatives of v evaluated in $\{l,a,\delta|q=q_{max}\}$[9]

$$\frac{\partial v}{\partial l} = 0 \tag{4.8}$$

$$\frac{\partial v}{\partial a} = v^2\{\rho-\rho_s-(1-a)(\frac{\alpha-\alpha_s}{\alpha_s})\frac{\partial \rho_s}{\partial a}\} \tag{4.9}$$

$$\frac{\partial v}{\partial \delta} = - v^2 \tag{4.10}$$

The first equation shows that managers select the same technique of production as shareholders.[10] Both the other first-order conditions are, however, generally not satisfied. This means that managers will pursue a different strategy towards growth, debt and pay-out. Equation (4.10) implies that the partial derivative with respect to δ is always negative ($\partial v/\partial \delta<0$) irrespective of the debt ratio a. For maximization of v managers will therefore always choose a lower pay-out than shareholders.

With regard to the debt ratio the outcome is less clear-cut. Equation (4.9) implies that managers select a higher debt ratio ($\partial v/\partial a>0$) if they have a higher discount rate than shareholders ($\rho>\rho_s$), or if they are more risk averse ($\alpha>\alpha_s$). However, if managers have a lower time preference and are more risk averse than shareholders, they will prefer a lower debt ratio than shareholders.

As the growth rate depends on both the debt ratio and the pay-out rate it cannot be unambiguously determined whether managers do desire a higher growth rate than shareholders. The positive effect of the lower pay-out rate may in principle be offset by a reduction in the debt ratio. However, as we have seen this is only possible in the case of a very conservative management, that is, if managers are more risk averse or have a lower discount rate than shareholders. Given the enterprising nature of managers this possibility seems hardly likely. Therefore, it can be concluded with reasonable confidence that the management realizes a higher growth rate than that

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[9] See Appendix 4.B for the derivation of these results.

[10] This conclusion is only valid if managers measure the size of the firm by its capital stock. If, however, managers maximize production rather than capital stock it is obvious that they will choose a more labour intensive technique, with a higher production per unit of capital (see also section 3.6)

Table 4.2 Corporate strategy

	normal management ($\rho \geq \rho_s$, $\alpha \leq \alpha_s$)	conservative management ($\rho < \rho_s$, $\alpha > \alpha_s$)
choice of technique	$l_{vmax} = l_{qmax}$	$l_{vmax} = l_{qmax}$
pay-out of profits	$\delta_{vmax} < \delta_{qmax}$	$\delta_{vmax} < \delta_{qmax}$
debt ratio	$a_{vmax} \geq a_{qmax}$	$a_{vmax} \gtrless a_{qmax}$
growth rate	$i_{vmax} \geq i_{qmax}$	$i_{vmax} \gtrless i_{qmax}$

vmax = corporate strategy; qmax = optimum strategy for shareholders

preferred by shareholders.

The differences between the corporate strategy (maximizing v) and the optimum strategy for shareholders (maximizing q) are summarized in *Table 4.2*. The size of the divergence depends largely on the ownership structure of the firm reflected by the σ function. Differences in strategy will be smaller if the chance of dismissal when $q < q(max)$ is greater. This is corroborated by the numerical results of *table 4.3*. This table is based on the following explicit functions for σ_s and σ:

$$\sigma_s = \nu.e^{-\omega\delta} \tag{4.11}$$

$$\sigma = \theta(-1+e^{\varphi(q(max)-q)/q}) \tag{4.12}$$

These functions satisfy the theoretical requirements of the implicit functions given above (eqs. 4.1 and 4.3). In the second equation the chance of dismissal varies with φ. If $\varphi=\infty$, the risk of dismissal is infinite for any $q < q_{max}$. In this case shareholders effectively have full control over the firm and will thus enforce a strategy that maximizes q. In the numerical example presented in the table this entails a choice of $i=-0.4\%$ and $\delta=1.01$ yielding a maximum q of 1.40. If φ becomes smaller, the influence of shareholders weakens which is reflected in a lower pay-out rate, higher growth rate and higher debt ratio. In the extreme if $\varphi=0$ the risk of dismissal is nil ($\sigma=0$) for any q. In this hypothetical case the management can freely lower the pay-out and raise the floatation of equity without worrying about the market valuation and the continuity of their jobs. In this case, however, no interior solution for δ and the growth rate can be found, as managers can always find a policy where $i>\rho$, so that the optimization is inevitably subject to the growth-stock paradox.

Table 4.3 Some numerical results for the corporate firm

φ	v	q	i(%)	δ	a
∞	5.70	1.40	-0.4	0.10	0.12
50	6.70	1.38	2.1	0.08	0.26
20	7.49	1.33	3.9	0.07	0.32
10	8.42	1.26	5.9	0.06	0.37

Explanation: π=0.10; r=0.02; η=0.025: η_s=0.02; θ=0.01; φ=20; ν=0.1; ω=15; $\alpha=\alpha_s$=10; var(π)=0.006; var(r)=0.005; covar(π,r)=0.002.

4.3 COSTS OF GROWTH

Managerial theories of growth traditionally explain the restraint on the firm's growth by declining profitability rather than by increasing financial risk. In this section we shall investigate the consequences for the determination of growth and finance when a negative growth-profitability relation is incorporated in our basic model. As mentioned in section 3.2 the growth-profitability frontier has been motivated in basically two ways: the *internal* approach, inspired by Penrose (1959), which emphasizes the organizational costs related to the expansion of the firm, and the *external* approach which follows Marris (1964) and concentrates on the product-market constraints and the 'demand shifting' effort necessary to maintain a certain growth of sales.

Annex 4.I to this chapter analyses both aspects on the basis of an integrated model of production, marketing and growth. This model takes account of the labour necessary for production as well as managerial labour and marketing effort. One of the interesting aspects of this analysis concerns the simultaneity of decisions concerning technique of production, marketing effort and the rate of investment. With regard to the present analysis we shall, however, skip the modelling of underlying marketing-, organization- and production-decisions and start directly from the growth-profitability frontier which emerges from this analysis. As is shown in the annex, the profit rate of the firm is related negatively to the rate of growth (i), the wage rate (W), the depreciation rate (ψ) and the tax rate on profits (τ_π) and positively to the volume of market demand (Z) and market demand growth ($\hat{Z}$), thus

$$\pi = \pi(i, W, \psi, \tau_\pi, Z, \hat{Z}) \quad \pi_i,\pi_W,\pi_3,\pi_\tau<0;\ \pi_Z,\pi_{\hat{Z}}>0 \tag{4.13}$$

where $\hat{Z}$ = DZ/Z. The negative impact of i on π is due to the marketing and organizational effort attached to growth[11]. The effects of W, ψ and τ_π are evident. Market demand and its growth have a positive impact because ample markets make sales more easy, and thus reduce marketing costs. The consequences of this function for the firm's strategy and the conflict between managers and shareholders will be discussed with respect to the basic model described in the foregoing chapter.

Optimum growth rate

First, it can be noticed that at a given level of the profit rate the negative π-i relation leads to a lower optimum growth rate. In comparison with the 'old' first derivative with respect to i (eq. 3.16), the first order condition for optimum growth rate is now:

$$\frac{dv}{di} = \frac{dv}{di}(\text{old}) + \frac{\partial v}{\partial \pi} . \pi_i = 0 \tag{4.14}$$

Since $\partial v/\partial \pi > 0$ and $\pi_i < 0$ the marginal effect of the growth rate on v is always negative at the old optimum, where of course dv/di(old)=0. Therefore, it can be concluded that in the presence of costs of growth the model still yields a finite optimum for the growth rate, which is lower as the slope of the π-i frontier is steeper.

Conflicting strategies

Now let us consider the difference in desired growth rate between managers and shareholders. In chapter 3 it was established that if managers maximize the capital stock there was no essential conflict between managers and shareholders regarding the investment-risk trade-off, except due to differences in time preference or risk aversion. The introduction of a growth-profitability frontier gives rise, however, to a more fundamental conflict of interests. Since in the steady state $v = q/\delta$ (eqs. 4.2 and 4.4), the first order condition for v can be written as

$$\frac{dv}{di} = \frac{1}{\delta}\left(\frac{dq}{di} - v\delta\pi_i\right) \tag{4.15}$$

This expression shows that in the shareholder's optimum $q = q_{max}$ (where dq/di=0) the growth rate still has a positive effect on v (dv/di>0). Therefore managers will choose a higher growth rate, thus

$$i_{vmax} > i_{qmax} \qquad \text{for any } \pi_i < 0$$

---

[11] As rapid shrinkage of the firm requires great organizational effort, just as fast expansion, it seems likely that the growth-profitability relation is positive for i<<0. For the moment, however, we shall neglect this possibility, and concentrate on the downward sloping segment of the π-i frontier.

and hence for any given δ

$$a_{vmax} > a_{qmax}\ ;\ K(0)_{vmax} > K(0)_{qmax}\ ;\ \pi_{vmax} < \pi_{qmax}$$

4.4 THE ADJUSTMENT PROCESS

So far we have concentrated on the optimum growth path under the restriction of a once and for all constant growth rate. This steady growth approach was warranted in the simple model where the capital stock could be varied instantaneously and without any cost. However, after the incorporation of the Penrose- and Marris-effects this approach is no longer warranted. One cannot on the one hand assume that growth of capital and production requires organizational and marketing costs, and at the same time let the initial stock of capital be varied freely. Note that this is not always recognized properly in models of corporate growth (e.g. Marris 1971, Slater 1980, Odagiri 1981).

For this reason we shall in this section drop the assumptions of a once and for all constant growth rate and a free initial stock of capital, and return to the general formulation of the optimization problem for the fully equity rationed firm (eq. 3.9a in the foregoing chapter). After substitution for q this can, for a given pay-out rate, be written as

$$\underset{i}{\text{Maximize}}\ v = \frac{1}{V_o} \int_0^\infty K(0) \exp[\int_0^t \{i_\tau - \tfrac{1}{2}\alpha \mathrm{var}(i)_\tau - \eta\}\, d\tau]\, dt \tag{4.16}$$

Unfortunately the mathematical form of this equation is too complex to handle on the analytical level. However, in order to be able to get an impression of the adjustment process we shall analyse the following simplified variant which is more familiar in control theory.

$$\underset{i}{\text{Maximize}} \int_0^\infty [\{i_t - \tfrac{1}{2}\alpha \mathrm{var}(i)_t\}\, e^{-\eta t}]\, dt \tag{4.17}$$

subject to

$$\pi = \pi_o - \chi(i-i_o)^2 \qquad\qquad \chi>0 \tag{4.18}$$

where var(i) is given by the known function of the debt ratio a (eq. 3.8) and the change of a follows from the budget constraint (3.5). According to (4.17) managers maximize the certainty equivalent of the *growth* of the firm, rather than the discounted *size*. Since the initial size of the firm is given this alternative representation is not fundamentally different from the original model. The basic considerations regarding the time path of i are similar; the trade-off between growth and (future) risk is still the central relationship.[12] As will be argued below, most of the conclusions based on the simplified version are valid for the original model as well. However, as the timing of the growth-risk trade-off is different, the precise shape of the adjustment trajectory will be different.

Equation (4.18) represents a simple version of the growth-profitability frontier. All determinants of π other than i are included in the autonomous factor π_o. The quadratic shape of this function can be motivated by the observation that not only the rapid growth but also the rapid shrinkage of the firm requires large organizational costs.[13] According to equation (4.18) the costs of growth decline when $i<i_o$ and rise when $i>i_o$. The costs of growth are thus minimal at $i = i_o$; that is if the firm shrinks at the rate i_o (we assume $i_o \leq 0$).

On the basis of these functions the optimum time path can be established from the (adjusted) Hamiltonian system (suppressing the time subscripts)

$$H^* = i - \tfrac{1}{2}\alpha \, var(i) - \lambda[\{\pi_o - \delta - \chi(i-i_o)^2\} - ra - (1-a)i] \tag{4.19}$$

$$a(0) = a_o$$

$$\lim_{t\to\infty} e^{-\eta t} \lambda(t).a(t) = 0$$

The first order conditions are

$$\frac{dH^*}{di} = 0; \quad \frac{dH^*}{da} = \eta\lambda - D\lambda \quad \text{and} \quad \frac{dH^*}{d\lambda} = Da \tag{4.20}$$

which implies

$$\begin{aligned} &1 + \lambda\{(1-a) + 2\chi(1-\delta')(i-i_o)\} = 0 \\ &\tfrac{1}{2}\alpha \frac{d var(i)}{da} + \lambda(i-r) = D\lambda - \eta\lambda \end{aligned} \tag{4.21}$$

where δ' ($=\partial\delta/\partial\pi$) stands for effect of total profits on the pay-out. These conditions

---

[12] Under the restriction of a given initial size and constant growth rate, both formulations also yield the same solution, namely the growth rate that maximizes the risk-discounted growth rate, $\{i-\tfrac{1}{2}\alpha \, var(i)\}$.

[13] See also footnote 11 above.

can be resolved into the following differential equation for i

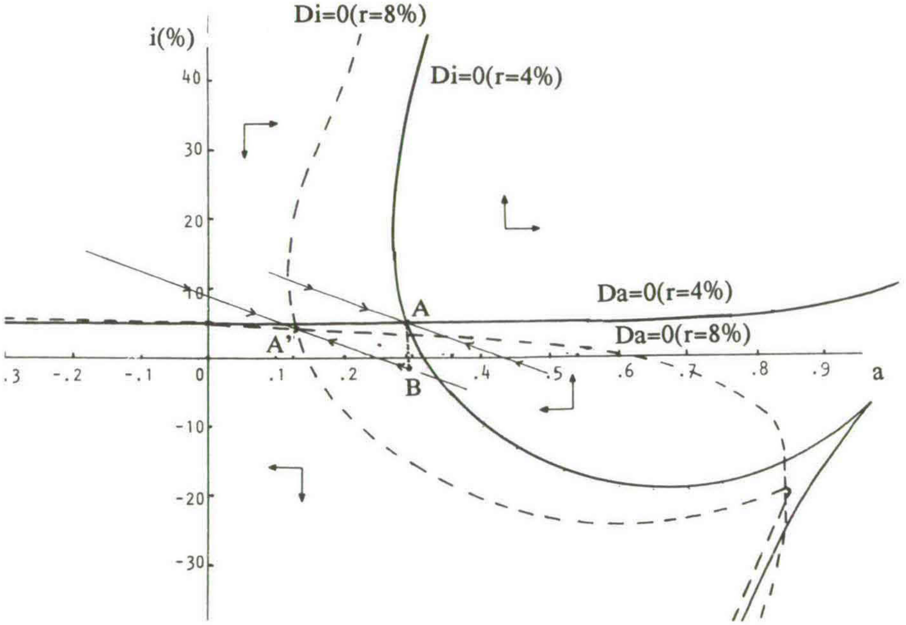
$$Di = \frac{1}{2\chi(1-\delta)} [r - \eta(1-a) - \{\pi_o - \delta - \chi(i-i_o)^2\} - 2\chi(1-\delta')(i-i_o)(i+\eta-r) + \alpha\{(1-a) + 2\chi(1-\delta')(i-i_o)\}^2.\{var(\pi)+a.var(r)-(1+a)covar(\pi,r)\}(1-a)^{-3}] \quad (4.22)$$

Together with the budget constraint (3.5) this equation determines the dynamics of i and a.

Debtor firm

Although this differential system cannot be solved explicitly it can be analysed qualitatively by means of the phase diagram (*figure 4.3*) which shows the Di=0 and Da=0 curves in the {i,a} plane. Because of the quadratic shape of the i-π frontier more than one solution may exist, but there is only one optimum solution A (i=4.9%, a=0.29) characterized by the familiar saddle-point configuration. The dashed line shows the hypothetical adjustment trajectory towards this optimum.

Figure 4.3 Phase diagram for a debtor firm



Explanation: π_o=0.10; χ=1; i_o=-0.04; η=0.02 and all other parameters as in table 4.3.

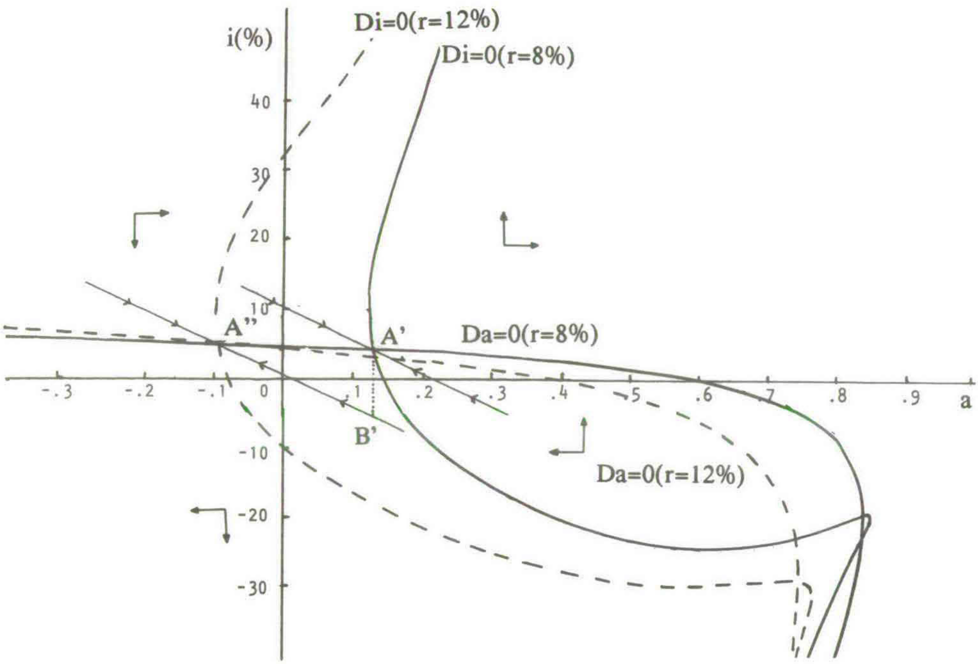
The same figure also shows the Di=0 and Da=0 curves for a higher interest rate (8% instead of 4%) yielding a new optimum A' at a lower growth rate and a smaller debt ratio (i=4.2%, a=0.12). It can be seen from the figure that the impact effect of the increase in r is greater than its ultimate effect. After the rise in r, the growth rate falls to a point B on the lower adjustment trajectory, whereafter it gradually increases again as the debt ratio tends to its new, lower steady state optimum.

The difference between the impact effect and the ultimate effect depends on the slope of the π-i frontier. If π is very sensitive to changes in i (χ large) it is very 'expensive' to vary i much along the adjustment trajectory. Thus the optimum trajectory will be flat. As a corollary the adjustment process will also take longer. As χ becomes smaller the initial drop in i becomes sharper, and the adjustment time shorter. If $\chi \downarrow 0$ the speed of adjustment becomes infinitely large.

Creditor firm

In the above case both the impact effect and the ultimate effect of a higher interest rate on the growth rate are negative. As we have seen in chapter 3 this is not

Figure 4.4 Phase diagram for a creditor firm



Explanation: see figure 4.3

Figure 4.5 A rise in interest rate: time path of K, i and a

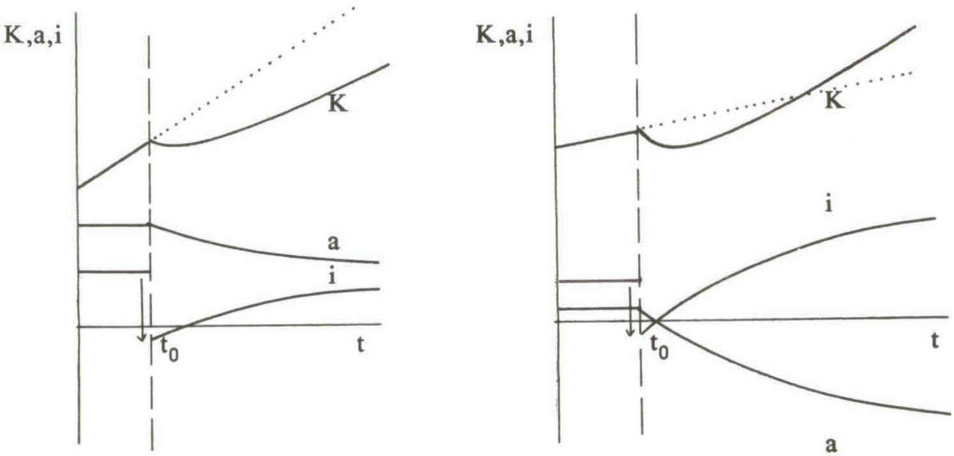


fig.a debtor firm **fig.b** creditor firm

necessarily so. When the firm is a net creditor (a<0) and π is low in relation to r, the steady state effect of the rise in r may be *reversed*, thus yielding a higher growth rate.

This is illustrated in *figure 4.4* showing the adjustment trajectory when the interest rate rises further from 8% to 12%. The new steady state position at r=12% is now characterized by a *higher* growth rate and a lower debt ratio (i=5.3%, a=-0.10). This new equilibrium can, however, only be reached by temporarily reducing the growth rate in order to let the rate of indebtedness decrease. After the discrete reduction in the growth rate, it will gradually recover again as debt declines over time. Ultimately i will even rise above its original level. Hence, in this case the higher interest leads to a *lower* growth rate in the short term and a *higher* growth rate in the long term.

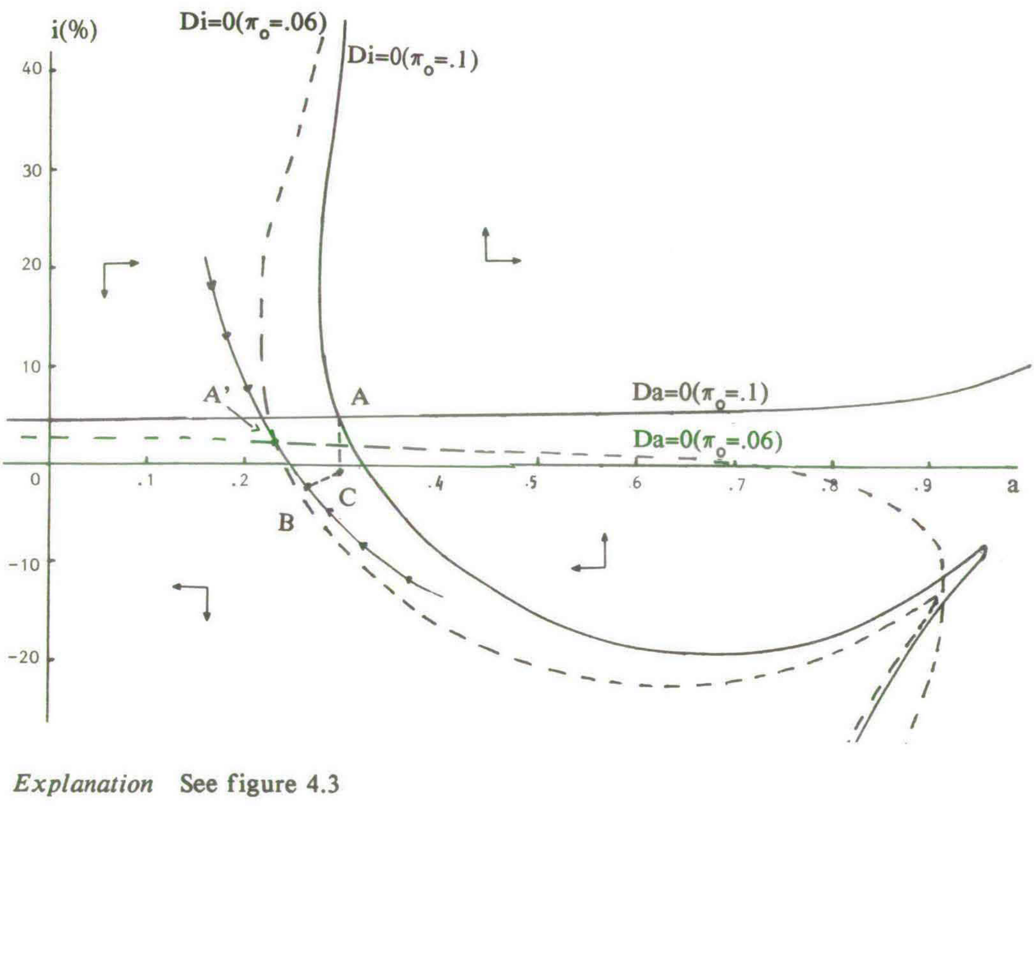
The impact and ultimate effects of the higher interest rate for the debtor and the creditor firm are summarized in *figure 4.5*. This figure shows the time paths of the capital stock, the growth rate and the debt ratio after a permanent rise in the interest rate at time t_0. *Fig.a* refers to the 'normal' case with negative impact and ultimate effects and *fig.b* to the 'perverse' case with negative impact effect and positive ultimate effect.

Fall in profitability

One of the characteristics which clearly emerges from *figures 4.3* to *4.5* is the 'overshooting' of the adjustment process. Similar overshooting processes may occur after exogenous changes in the profitability of investment (π_0) or the variability of the profit rate var(π) or the interest rate var(r). As a final example we shall therefore consider a shift in profitability. *Figure 4.6* shows that a fall in π_0 from 10% to 6% eventually leads - after a similar overshooting process -to a lower steady state debt ratio (a=0.23) and a lower growth rate (i=2.4%).

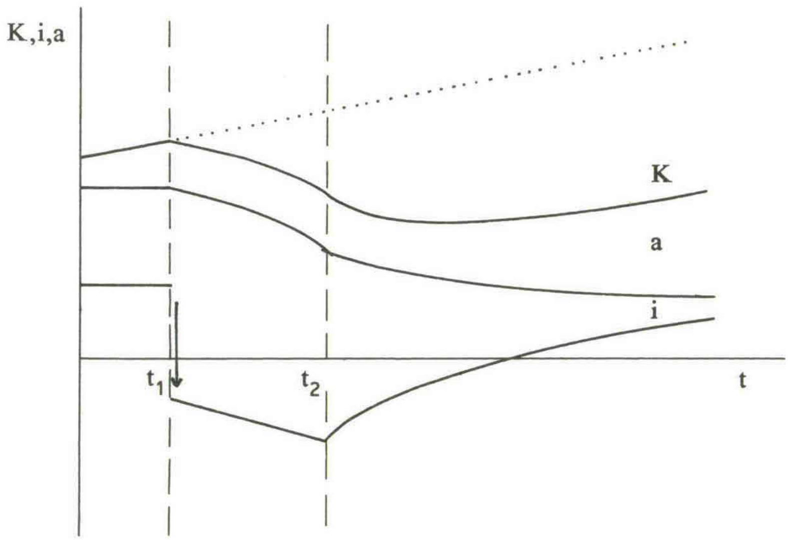
In addition this figure shows the trajectory when the fall in profitability is *foreseen* by entrepreneurs at an earlier moment. This may happen, for instance, if it is announced at time t_1 that wages, or taxes, are to be raised at t_2 ($>t_1$). In this case the firm will already reduce its growth at the moment of the announcement (t_1). This is shown in the figure by the discrete fall to point C; then the growth rate gradually falls further until the actual fall in profit rate at t_2. From that moment onwards the growth rate starts rising again along the trajectory B-A'. Thus, as soon as the future fall in profits becomes known, the growth is reduced immediately in order to achieve a better point of departure, i.e. a lower debt ratio, before profits actually fall. This is illustrated in *figure 4.7*.

Figure 4.6 A (foreseen) fall in profitability



Explanation See figure 4.3

Figure 4.7 A foreseen fall in π: time paths for K, i, a



These conclusions based on the simplified model will not really be different from the conclusions which can be drawn on the basis of the original version of the optimization problem. The ultimate effects on the growth rate found above correspond to the conclusions from the steady state analysis in the foregoing chapter, for the debtor firm as well as for the creditor firm. As to the impact effects it can be seen that their direction must be the same for the original model as well[14]; an ultimate decrease in the debt ratio will always require a (temporary) reduction of the growth rate, and an ultimate rise in debt an increase in i.

The dynamics considered above emerge from the fact that the firm's debt ratio can only be adjusted slowly. This follows from the assumptions of a given initial stock of capital and given initial net worth of the firm. In the above model the firm has been assumed to have no access to the equity market at all. This is, of course, relevant for many firms in reality. The analysis may, however, be generalized for the case of a big corporation which can raise funds by floating equity as well. It is evident that if the

[14] This follows from the fact that the optimum must be characterized by a saddle-point configuration and that the Da function is, of course, identical in both the formulations of the optimization problem. As can be seen from the phase diagrams above the adjustment trajectory must therefore always lie under the Da=0 curve in the case of a reduction in a and above this curve in case of a rise in a.

firm can raise equity freely without any limitation, there no need for a delayed adjustment to the desired debt ratio. If, however, as we have argued in section 4.2, there is only a limited demand for the firm's shares, so that equity can only be raised at increasing costs, these costs may impose an effective constraint on the adjustment of the firm's financial position.[15] Then, the adjustment process will not be essentially different for a big corporation with access to the equity market than for a small firm without access as was considered above.

4.5 CONCLUSION

In this chapter we have elaborated the basic model of finance and growth with respect to three points: the introduction of the equity market, the incorporation of costs of growth, and the analysis of the adjustment process.

By the introduction of the equity market together with a hazard function for the risk of dismissal for managers we could establish the simultaneous equilibrium for the growth and pay-out policy of a big corporation. The analysis corroborates the basic conclusion of the foregoing chapter that when the supply of equity capital is limited (i.e. absent as in chapter 3, or subject to rising costs as in section 4.2), the principle of increasing risk provides an effective restraint on the firm's growth.

Introduction of the costs of growth did not significantly change the conclusions of the basic model. It sharpens, however, the conflict between management and shareholders, as the pursuit of rapid growth now directly affects the profit rate.

One of the central elements of our modelling of the growth of the firm has been the conflict of interests between shareholders and the management. The main differences in strategy between a firm that maximizes market valuation (q) and a firm that maximizes the discounted size (v) are summarized in the scheme below. By way of reference the results of Williamson (1966) and Seoka (1985) are also given. These results show that managers generally aim at a higher growth rate and lower pay-out rate than shareholders. Only if managers are significantly more risk averse and have a higher discount rate than shareholders, is it not certain that they would choose a higher growth rate. In these circumstances they may prefer a lower debt ratio. In the case of a falling growth-profitability relation, or if managers maximize production (or sales) rather than the capital stock, the firm's strategy will deviate from instant profit-maximizing, which is desired by shareholders.

[15] If the costs of raising equity are related to the flow of new ('outside', see section 3.3) equity, these costs may put a similar restraint on the adjustment of the firm's net worth, as the adjustment costs of investment prevents discrete changes in capital stock. A motivation for the relation between costs and the floatation of shares can be found in Jensen and Meckling (1976), who assume that agency costs rise with the amount of outside finance.

Differences between a v-maximizing firm and a q-maximizing firm
(+ indicating a positive difference)

	i	δ	a	K(0)	y	π
small firm (π_i=0)						
- capital-maximizing	+	-[1])	0	0	0	0
- production maximizing	+	-[1])	0	0	+	-
small firm (π_i<0)	+	-	+	+	?	-
large firm (π_i=0)						
- normal management	+	-	+	+	0	0
- conservative management	?	-	?	?	0	0
Williamson (1966)	?	?	.	+	+	-
Seoka (1985)	+	.	.	+	.	-

[1]) These results are valid only if the discount rate of shareholders exceeds the net rate of return of the firm, so that shareholders desire $\delta=\infty$ and $i=-\infty$.

Consistent modelling requires that if one allows for costs of growth, one should also drop the assumption of a free initial size. Analysis of the adjustment process revealed that changes in profit rates and interest rates may lead to significant changes in investment, in the short term as well as the long term. In the presence of imperfect equity markets changes in profits and interest rates affect investment in two ways: through the availability of internal finance and through the change of the desired financial structure. The first, 'internal savings,' effect is essentially a short-term effect, as it implies direct adjustment of investment to the flow of internal finance.[16] The second effect may, however, produce quite persistent changes in investment. This is because the firm's balance sheet can only be adjusted slowly to the desired new composition. The combination of both effects means that the adjustment process is often characterized by *overshooting*; that is, the impact effect of an exogenous change proves to be stronger than the ultimate effect. For a creditor firm the impact and ultimate effects may even have different signs. This corroborates our observation in chapter 3 that the interest effect on investment may be *reversed* for a creditor firm.

These dynamic processes may provide an insight into the actual economic

[16] The recognition of financing constraints has given new support to the 'internal finance' view of investment. From their empirical research on U.S. corporate investment Fazzari, Hubbard and Petersen (1988,p.185) thus conclude that "financing constraints could account for a large proportion of the aggregate variability of investment."

developments in the most recent period. This period was characterized by large aggregate shocks which had a strong impact on the financial structure of firms. It is well-established - at least for European economies - that the adverse shift in profits and risk in the 1970's caused a long process of adjustment before the discrepancy between the actual and the desired financial structure was resolved. As changes in the investment behaviour of firms have great consequences for macroeconomic aggregates such as effective demand and employment, which in turn have great consequences for the strategy of firms, this adjustment process should not be considered in isolation. In the next chapters we shall therefore return to the macroeconomic level, and analyse the dynamic interaction between firms, government and workers for the medium period (chapter 5), the long period (chapter 6) and the open economy (chapter 7).

ANNEX 4.I A DIGRESSION ON COSTS OF GROWTH

This annex establishes the growth-profitability (π-i) frontier on the basis of an integrated model of production, sales effort and growth. In this analysis the growth rate is taken as given; the consequences of the π-i frontier for the dynamic optimum of the firm are discussed in section 4.3 of this chapter.

Unlike most models of corporate growth we shall take explicit account of the simultaneity of decisions on technique of production, marketing effort and growth strategy.[17] The model incorporates both the *Penrose relation* between growth and the need for managerial services and the *Marris relation* between growth and 'demand shifting' effort. As production is measured by value added, the costs of growth should be expressed in additional labour required or lower productivity of the capital stock. Adopting the first possibility, the Marris and Penrose effects can be incorporated in the production model as follows:

$$y = y(l_1) \qquad y'>0;\ y''<0 \tag{I.1}$$

$$l_2 = (1/K).l(\hat{G}, G, Z) \qquad l_{\hat{G}}>0;\ l_G>0;\ l_Z<0;\ l_{\hat{G}G}<0 \tag{I.2}$$

$$l_3 = l_3(i) \qquad l_i>0 \text{ for any } i>0 \tag{I.3}$$

$$l = l_1 + l_2 + l_3 \tag{I.4}$$

$$y^d = (1/K).y^d(P, G, Z) \qquad y_P<0;\ y_G, y_Z>0 \tag{I.5}$$

$$y = y^d \tag{I.6}$$

[17] In most previous models the technique of production is assumed to be exogenous (cf. Solow 1971, Auberada 1979, Seoka 1985) or at least to be independent of the choice of marketing effort and the managerial costs of growth (Marris 1971, Odagiri 1981). For an interesting attempt to integrate the choice of technique and the managerial costs of growth, see Slater (1980).

where l_1 = production labour
l_2 = marketing labour
l_3 = organization labour
G = stock of goodwill
$\hat{G}$ = DG/G
y^d = demand for the firm's products
Z = total real demand on the product market

The lower case symbols l_1, l_2, l_3 and y^d are again expressed as ratios to the firm's capital stock. The first equation is the familiar production function, now however not with total employment as argument, but only labour as far as it is involved in the production process (l_1). Equation (I.2) represents the Marris relation and equation (I.3) the Penrose relation. Following Solow (1971) and Seoka (1985) marketing effort (l_2) is related to the growth of the stock of goodwill (G).[18] As it is plausible that it becomes increasingly difficult to expand the stock of goodwill as it becomes greater we have also included G in this function (l_G>0). Finally, the state variable Z is also included as it seems plausible that it is easier to increase goodwill as the market is larger in relation to the present stock of goodwill (hence l_Z<0).

The Penrose relation between investment and (labour) costs (eq.I.3) resembles the well-known 'adjustment cost' function in neoclassical investment theory, but unlike this function the Penrose relation is not required to be convex in i.[19] Equation (I.4) defines total demand for labour as the sum of production, marketing and organization labour. Note that in this modelling of demand for labour it is implicitly assumed that (managerial) labour is freely available and can be absorbed without any cost.[20] According to equation (I.5) demand for the firm's products depends on the stock of goodwill, the price set by the firm and total market demand.[21] Equation (I.6) finally defines equilibrium between production and sales of products.

If the management maximizes the discounted size of the firm's capital stock, it must at any moment select the technique of production l_1, marketing effort l_2 and expansion effort l_3 that maximizes the profit rate[22]

$$\pi = Py - W(l_1+l_2+l_3) \tag{I.7}$$

[18] Seoka (1985) has pointed out that it is most appropriate to relate marketing effort to the rate of change of G.

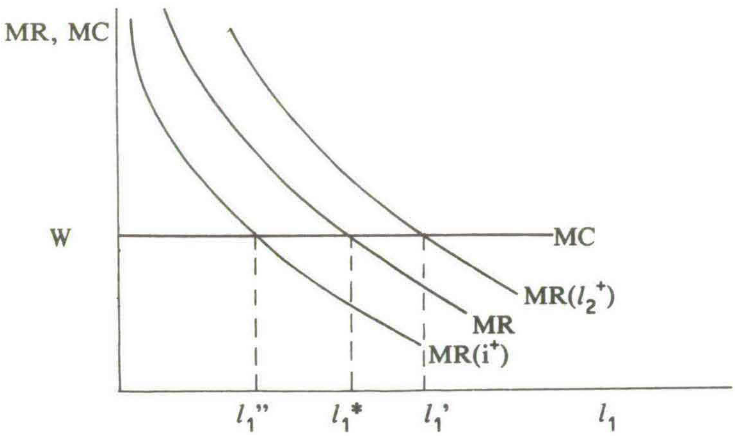
[19] See e.g. Nickell(1978) for a critique on this convexity assumption.

[20] For alternative assumptions see Slater (1980) and Moss (1984).

[21] This latter factor, which determines the locus of the demand curve, is systematically neglected in most managerial theories. Williamson (1966, p.4) motivates this neglect arguing that "virtually unlimited opportunities for diversification remove any necessity .. to move down existing demand curves".

[22] If managers aim at maximum production rather than capital stock the momentary equilibrium will of course be different. For brevity, we shall not examine this case here.

Figure I.1 Optimum technique of production



where K, Z, $\hat{Z}$ and W are given. The initial stock of goodwill G is assumed to be a free variable.[23]

First notice that transformation of equation (I.5) in growth rates yields the following condition for the growth of the stock of goodwill.

$$\hat{G} = \frac{1}{\epsilon_G}\left(i - \epsilon_P \frac{DP}{P} - \epsilon_Z \frac{DZ}{Z}\right) \qquad \epsilon_G>0;\ \epsilon_P<0;\ \epsilon_Z>0 \tag{I.8}$$

where $\epsilon_P = y_P.(P/yK)$ denotes the price-elasticity of demand and $\epsilon_G = y_G.(G/yK)$ its elasticity with respect to goodwill. As prices cannot be lowered for ever this equation implies a unique relation between the growth rate of goodwill ($\hat{G}$) and the growth rate of capital stock (i) and market demand rate (DZ/Z).[24] After substitution of this

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[23] The assumption of free G(0) is necessary for expository reasons, because with a fixed initial state the steady state path would no longer represent a dynamic optimum. As a motivation for the free G(0) one may imagine that the firm chooses its initial K(0) as well as G(0) by the take-over of existing firms together with their stock of goodwill.

[24] For the steady state to exist for $i \neq DZ/Z$, in fact very special conditions must be satisfied as to the elasticities in the functions for demand (I.5) and marketing effort (I.2). Time differentiation of these equations gives respectively

$$i + Dy/y = \epsilon_P\, DP/P + \epsilon_G\, DG/G + \epsilon_Z\, DZ/Z$$

$$i + Dl_2/l_2 = \lambda_{\hat{G}}\, D\hat{G}/\hat{G} + \lambda_G\, DG/G + \lambda_Z\, DZ/Z$$

where λ_j stands for the elasticity of l_2 with respect to variable j. Since in the steady state DP, Dy, Dl_2, $D\hat{G}=0$ these equations are inconsistent unless $\lambda_G=\epsilon_G$ and $\lambda_Z=\epsilon_Z$, which means that a change in G should lead to an equiproportional change in demand (y) and marketing labour (l_2). It may be noted that Williamson (1966), Solow (1971), Auberada (1979), Odagiri (1981) and others introduce this assumption implicitly by disregarding the influence of Z altogether (hence $\epsilon_Z=\lambda_Z=0$) and postulating linear homogeneity of equations I.2 and I.5 (hence $\lambda_G=\epsilon_G=1$). In that case equation I.8 reduces to the simple equality between the growth rate of capital stock and the growth rate of goodwill, $i=\hat{G}$.

relation in I.2. we find for the first order conditions for maximum profit rate

$$\frac{\partial\pi}{\partial l_1} = (\frac{1}{\epsilon_P}+1)P.y' - W = 0 \tag{I.9}$$

$$\frac{\partial\pi}{\partial l_2} = - \frac{\epsilon_G}{\epsilon_P}\,\frac{1}{l_G}\,\frac{Py}{G} - W = 0 \tag{I.10}$$

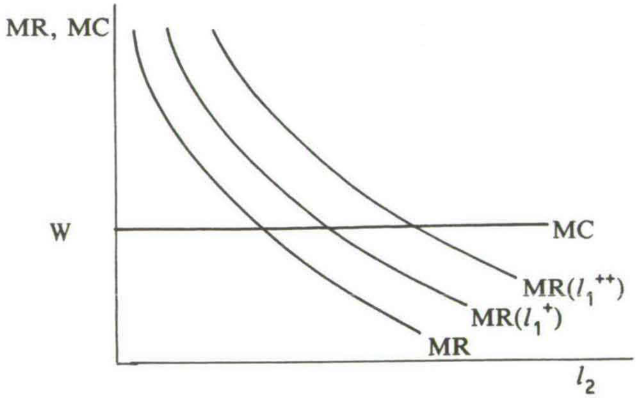
In both conditions the first term may be considered as the marginal returns (MR) of labour (l_1 and l_2) and the second term, the wage rate W, as the marginal cost (MC). Provided that the price-elasticity of demand ϵ_P is less than -1 the first condition yields the optimum for l_1. This is shown in *figure I.1* by the intersection A of the MC and the MR schedules.

Now consider an increase in marketing effort l_2. Through the larger l_2 the firm can afford a greater initial stock of goodwill at the given rate of growth $\hat{G}$, and thanks to this greater G(0) it can earn a higher price on its sales. Because the marginal return of production labour is thus increased, the firm will change its technique of production and employ more production labourers (from l_1^* to $l_1^{*\prime}$). A greater marketing effort thus leads to a more labour intensive technique of production,

$$\partial l_1^*/\partial l_2 > 0$$

The optimum for marketing effort l_2 can be explained with reference to *figure I.2.* The marginal costs are, of course, again given by the wage rate W. The marginal returns of l_2 result from the fact that the firm can maintain a greater initial stock of goodwill G(O) as l_2 is larger, and thus ask a higher price for its products. These marginal returns will, however, diminish as l_2 becomes larger. This implies a falling MR schedule in *figure I.2.*

Figure I.2 Optimum marketing effort



Now consider the effect of additional production labour on the optimum for marketing effort. At a given stock of goodwill the marginal returns of marketing labour will be larger as production is larger, and thus goodwill is smaller in relation to production and sales. Therefore, the increase in production labour (l_1) causes the MR schedule of l_2 in *figure I.2* to shift upward, thus leading to a larger optimum marketing effort. Hence

$$\partial l_2^* / \partial l_1 > 0$$

Bringing these results together we can establish the simultaneous optimum of l_1 and l_2. *Figure I.3* represents this optimum by the intersection of the l_1^* and l_2^* curves showing the mutual relationships between these variables established above.

Growth and profitability

In order to establish the relationship between growth and profitability we shall now consider the effect of a change in the growth rate i on this simultaneous production and marketing optimum. As we have seen a higher growth rate requires a greater marketing effort in order to realize a correspondingly higher growth of goodwill. Therefore, at a given input of marketing labour l_2, the firm can afford only a smaller initial stock of goodwill. Then, as marginal returns of production labour fall, the l_1 schedule in *figure I.1* shifts downward leading to the lower optimum of production labour. As for the production-marketing optimum represented in *figure I.3*, this manifests itself in a downward shift of the l_1^* schedule.

The smaller initial stock of goodwill changes the marginal returns of l_2 too. At a given volume of production labour it is evident that a smaller initial stock of goodwill implies the marginal returns of l_2 to be higher. Therefore the l_2^* curve in *figure I.3* shifts to the right.

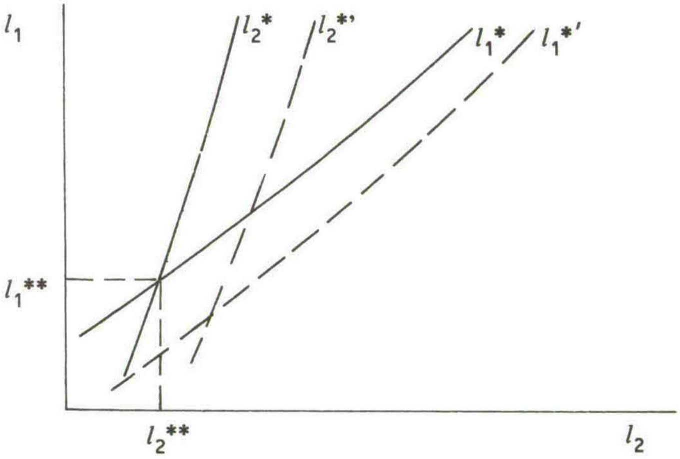
As both schedules in *figure I.3* shift to the right the overall result of a higher growth rate on the optimum $\{l_1^{**}, l_2^{**}\}$ cannot be readily assessed. It depends on the parameters of the model. It can, however, be shown that, under the plausible assumption that cross elasticities of l_1^* and l_2^* are not too great, so that the l_1^* curve is less steep than the l_2^* curve, the new equilibrium is characterized by a smaller optimum stock of goodwill G(0), and consequently a smaller volume of production y(0) and production labour l_1 (for a proof, see *Appendix 4.C*). This outcome is in accordance with intuition as a higher growth rate requires larger costs of maintaining a given stock of goodwill. If the firm therefore selects a smaller initial stock of goodwill, the marginal returns of production will fall, so that the new optimum for y and l_1 will be lower as well.

The change in optimum marketing effort (l_2) is ambiguous (see *Appendix 4.C*); on the one hand a higher growth rate requires a greater input of marketing labour, but on the other hand the smaller optimum stock of goodwill tends to reduce the need for l_2.

In summary the growth rate has the following effects on the optimum strategy

$$\frac{dy}{di} < 0; \quad \frac{dG(0)}{di} < 0; \quad \frac{dl_1}{di} < 0; \quad \frac{dl_2}{di} \gtrless 0; \quad \frac{dl_3}{di} > 0$$

Figure I.3 Simultaneous optimum for production and marketing



Finally, given this relation between optimum production and marketing effort and the growth rate, we are able to establish the determinants of the profit rate corresponding to this optimum. Writing π as a function of the given variables i, W, ψ, Z and its growth rate $\hat{Z}$ (=DZ/Z), we obtain

$$\pi = \pi(i, W, \psi, \tau_\pi, Z, \hat{Z}) \qquad \pi_i, \pi_W, \pi_3, \pi_\tau < 0, \ \pi_Z, \pi_{\hat{Z}} > 0 \tag{I.11}$$

Since in marketing-production optimum $\partial\pi/\partial l_1$ and $\partial\pi/\partial l_2$ are zero, it can be found for the partial derivatives

$$\pi_i = (1-\tau_\pi)\left(\frac{y}{y_P}\cdot\frac{l_{\hat{G}}}{l_G}\cdot\frac{yK}{G} - W.l_i\right) < 0$$

$$\pi_W = l_1 + l_2 + l_3 > 0$$

$$\pi_3 = 1 > 0$$

$$\pi_Z = (1-\tau_\pi)\left\{\frac{y}{y_P}.\left(\frac{l_Z}{l_G}\, y_G - y_Z\right)\right\} > 0$$

$$\pi_{\hat{Z}} = (1-\tau_\pi)\ \frac{y}{y_P}\cdot\frac{l_{\hat{G}}}{l_G}\cdot\frac{Z}{G} y_Z > 0$$

As to the signs of these derivatives it may be recalled that $y_P<0$ and $l_{\hat{Z}}<0$, and all other effects >0. Note that the growth rate has an unambiguously negative effect on the profit rate ($\pi_i<0$) because of the marketing cost (the first term) and the organizational cost (w.l_i). These effects represent the Marris and Penrose relations respectively. Further, the profit rate depends positively on the volume and growth of market demand and the constant wage, depreciation and tax rates. The consequences of this function to the firm's strategy are discussed in section 4.3.

Appendix 4.A SHAREHOLDERS' OPTIMUM

The optimum strategy from the point of view of shareholders is the strategy which maximizes the q ratio. Absent the growth stock paradox q can be written as

$$q = \frac{\delta}{\delta + (1-a)\rho - (\pi - ar)} \tag{A.1}$$

Given the relation for the discount rate (eq. 4.3) the first order conditions for l, a and δ are

$$\frac{\partial q}{\partial l} = \frac{\partial q}{\partial \pi}\frac{\partial \pi}{\partial l} = 0 \tag{A.2}$$

$$\frac{\partial q}{\partial a} = \frac{q^2}{\delta}\{\rho_s - r - (1-a)\frac{\partial \rho_s}{\partial a}\} = 0 \tag{A.3}$$

$$\frac{\partial q}{\partial \delta} = \frac{q^2}{\delta^2}\{(1-a)\rho_s - (\pi - ar) - (1-a)\delta\sigma_s'\} = 0 \tag{A.4}$$

The solution for i is implied by the simultaneous optimum for δ and a. The first condition (A.2) determines the optimum technique of production. The other conditions yield the optimum debt ratio and pay-out rate. If $\sigma_s' = 0$ the discount rate is independent of δ, so that A.3 determines the optimum of the debt ratio (a) independently of δ (see also note 26 in the foregoing chapter). However, in this case (or if $\sigma_s'' = 0$) the last condition A.4 cannot be fulfilled for any finite δ; hence $\sigma_s''>0$, is a necessary condition for an interior solution to exist for the optimum pay-out rate.

APPENDIX 4.B MANAGERIAL STRATEGY

The difference between the strategy of the firm and the desired strategy by shareholders can be established by comparing the first order conditions for v and those found for q in *Appendix 4.A*. The optimality conditions for v are

$$\frac{\partial v}{\partial l} = \frac{\partial v}{\partial \pi}\frac{\partial \pi}{\partial l} = 0 \tag{B.1}$$

$$\frac{\partial v}{\partial a} = v^2\{\rho - r - (1-a)\frac{\partial \rho}{\partial a}\} = 0 \tag{B.2}$$

$$\frac{\partial v}{\partial \delta} = -v^2\{(1-a)\frac{\partial \rho}{\partial \delta} + 1\} = 0 \tag{B.3}$$

These conditions shall now be evaluated in the optimum for q. First, notice that in this optimum

$$\frac{\partial \rho}{\partial \delta} = \frac{\partial q}{\partial \delta}\sigma' = 0 \quad \text{and} \quad \frac{\partial \rho}{\partial a} = \frac{\alpha}{\alpha_s}\frac{\partial \rho_s}{\partial a}$$

After substitution of the results for the maximum of q found in *Appendix 4.A*, the partial derivatives of v become

$$\frac{\partial v}{\partial l} = 0 \tag{B.4}$$

$$\frac{\partial v}{\partial a} = v^2\{\rho - \rho_s - (1-a)(\frac{\alpha-\alpha_s}{\alpha_s})\frac{\partial \rho_s}{\partial a}\} \tag{B.5}$$

$$\frac{\partial v}{\partial \delta} = -v^2 \tag{B.6}$$

These results are discussed in the text (section 4.2).

APPENDIX 4.C GROWTH AND PROFITABILITY

The effect of a change in the growth rate i on production and marketing equilibrium can be obtained from the total derivatives of the first order conditions I.2 and I.3.

$$\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} . \begin{bmatrix} dl_1 \\ dl_2 \end{bmatrix} + \begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix} . di = 0 \tag{C.1}$$

where $\pi_{hj}=\partial^2\pi/\partial l_h \partial l_j$ (h,j = 1,2) and $\pi_{hi}=\partial^2\pi/\partial l_h \partial i$ (i=growth rate). The determinant of the π-matrix is assumed to be positive, which corresponds to the assumption that the l_2^* curve in *figure I.3* is steeper than the l_1^* curve. For the direction of the effect of i on the equilibrium values of l_1 and l_2 equation C.1 implies:

$$\text{sign}\left(\frac{dl_1}{di}\right) = \text{sign}\,(\pi_{11}\pi_{2i} - \pi_{22}\pi_{1i}) \tag{C.2}$$

$$\text{sign}\left(\frac{dl_2}{di}\right) = \text{sign}\,(\pi_{21}\pi_{1i} - \pi_{11}\pi_{2i})$$

After substitution for π_{12} etc. it can be found that

$$\text{sign}\left(\frac{dl_1}{di}\right) = \text{sign}\,(\pi_{1G}.\pi_2 l_G . l_{G\hat{G}}) < 0 \tag{C.3}$$

$$\text{sign}\left(\frac{dl_2}{di}\right) = \text{sign}\{\pi_{1y}.\pi_{\hat{G}G} - \pi_{1G}.\pi_{2y}\} l_{\hat{G}} + \pi_{1y}.\pi_2 l_{\hat{G}}(l_{\hat{G}} - l_G . l_{G\hat{G}}) \gtrless 0$$

where π_{hj} is the partial derivative of the marginal return of l_h (h = 1,2) with respect to variable j (j = y,G,l_G). The influence of a higher i on the volume of production labour is unambiguously positive ($\partial l_1/\partial i > 0$), but the effect on marketing labour l_2 cannot be assessed with certainty. If the own second-order derivatives π_{1y} and π_{2G} are greater in absolute terms than the 'cross' derivatives π_{1G} and π_{2y} and if $l_{G\hat{G}}$ is smaller than $l_{\hat{G}}/l_G$, then the effect of i on yl_2 is unambiguously positive, but otherwise this effect is uncertain.

CHAPTER 5

THE KEYNESIAN CORRIDOR

5.1 INTRODUCTION

The next two chapters investigate the impact of corporate and public debt on the medium-term and long-term dynamics of a closed economy. The analysis develops the macroeconomic analysis of chapter 2 and the microeconomic analysis of corporate growth in chapters 3 and 4. In contrast with the 'steady state' analysis in chapter 2, in which an exogenous rate of growth and a constant interest and profit rate was assumed, the present analysis focusses on the underlying dynamics that should ensure steady state growth. Therefore we shall develop a model which incorporates a dynamic conception of investment behaviour, a monetary explanation of the interest rate, and a more sophisticated representation of fiscal policy.

The analysis is built up in two stages: the present chapter examines the medium period while the following chapter concentrates on the long period. The principal borderline between the medium and the long period concerns the conception of income-expenditure equilibrium. Following Malinvaud (1977) and Kuipers (1981) we assume that the medium period is characterized by a rigid technique of production and sluggish prices, and by sluggish expectations on prices and aggregate demand as well. As a result the disequilibrium dynamics of aggregate demand and production capacity are typical for the medium period. In the long period full capacity utilization is presupposed; here the analysis concentrates on the interaction between growth, income distribution, and the accumulation of debt and wealth.

The main objective of the medium-term analysis of the present chapter is to investigate whether the medium-period dynamics leads to a stable equilibrium with full capacity utilization, and what impact fiscal policy has on these dynamics. We start with a discussion of the model which serves as the basis for the medium period as well as for the long period analysis (section 5.2). Because of the prominent role of the government budget constraint in our analysis, the representation of fiscal policy will be discussed at some length in section 5.3. Next, in sections 5.4 - 5.7 the medium period model is elaborated and analysed with respect to its dynamic properties. Finally, section 5.8 makes an assessment of the impact of the policy regime on the stability of the system.

5.2 THE BASIC MODEL

The main features of the basic model are the following:

1. The economy is divided into three sectors: workers, the corporate sector and the government. *Workers* receive labour income (wages and unemployment benefits) as well as interest income on their holdings of government debt and corporate debt. The *corporate sector* encompasses all firms and their owners; this sector receives the profits after payment of debt service to the workers. In order to avoid the complications connected with the valuation of shares and capital goods (see section 2.3) it is assumed that workers do not possess shares; all shares are owned by the conglomerate of the entrepreneurs, (top)-managers and the large shareholders who control the corporate sector.[1] The third sector in this model is the *government* which raises income by taxing the income of workers and the corporate sector. The government spends its income on consumption, transfers and debt service.
2. Workers have a lower propensity to save from their income than members of the 'corporate class'. For both classes, consumption also depends on the interest rate and the amount of wealth.
3. Unlike the traditional 'Ricardian' post-Keynesian model considered in chapter 2, the rate of interest is not simply derived from the profit rate but determined - in a Keynesian fashion - by portfolio equilibrium. The portfolio consists of money and bonds only. There is no room for financial intermediaries; if they exist they are simply included in the corporate sector. Government debt and private debt are perfect substitutes. For simplicity, all money is assumed to be held by workers; firms are supposed to be able to hold all their liquidities in the form of interest-bearing assets.
4. In each model prices are sluggish in the sense that the level of prices is fixed at every instant, but that its rate of change (the inflation rate) may vary.

The model uses the following symbols:

a = debt of firms (net of holdings of government bonds)
b = government debt
c_j = propensity to consume of income for class j
C_j = total consumption of class j

[1] This division between workers and the corporate sector reconciles Kaldor's argument, that differential saving arises from the corporate structure of the modern economy, with Pasinetti's argument that workers also accumulate wealth. The basic problem with Kaldor's approach concerned the 'vanishing' of retained earnings (see 2.3). By taking shareholders and firms together we avoid this problem. Retained earnings are fully taken into account now, but because of the higher propensity to save of the 'corporate class', the impact of retained earnings on consumption is only small. This modelling is to be preferred to letting retained earnings vanish completely (cf. Chiang 1973, Darity 1980, Kuipers 1981).

g = government expenditure
i = net investment
l = employment (in efficiency units)
l_c = employment at full capacity utilization
l_s = labour supply (in efficiency units)
m = (base) money supply
m_d = (base) money demand
p = inflation
p_e = expected rate of inflation
π = profit rate
r = real interest rate (backward looking)
r_e = expected real interest rate (forward looking)
T_j = taxes of class j
τ_0 = tax rate on wage and transfer income
τ_1 = tax rate on interest income of workers
τ_2 = tax rate on net returns of the corporate sector
τ_3 = unemployment benefit
u = unemployment ($= l_s - l$)
w = sum of wages
y = production
y_c = productive capacity
y_g = income of the government sector
y_j = income after taxes of class j
z_j = wealth of class j

where j=1 for workers and j=2 for the corporate class. All stock and flow variables are expressed as ratios to capital stock. The basic equations of the model can be written as follows:

production and income

$$y_c = y(l_c) \qquad y'>0,\ y''<0 \tag{5.1}$$

$$y = w + \pi \tag{5.2}$$

$$y_1 = w + \tau_3 u + r(a+b) - pm - T_1 \qquad \tau_3 \geq 0 \tag{5.3}$$

$$y_2 = \pi - ar - T_2 \tag{5.4}$$

$$y_g = T_1 + T_2 - \tau_3 u - rb + pm \tag{5.5}$$

expenditure relations

$$y = C_1 + C_2 + g + i \tag{5.6}$$

$$C_1 = C_1(y_1,(1-\tau_1)r_e,z_1) \qquad 0<c_1<1;\ c_{1r}<0;\ c_{1z}<0 \tag{5.7}$$

$$C_2 = C_2(y_2,(1-\tau_2)r_e,z_2) \qquad 0<c_2<1;\ c_{2r}<0;\ c_{2z}<0 \tag{5.8}$$

$$T_1 = \tau_0\{w+\tau_3 u\} + \tau_1\{r(a+b)-pm\} \qquad 0 \leq \tau_0,\tau_1 \leq 1 \tag{5.9}$$

$$T_2 = \tau_2(\pi-ar) \qquad 0 \leq \tau_2 \leq 1 \tag{5.10}$$

$$g = g_o + \gamma_1\{T_1+T_2-\tau_3 u\} - \gamma_2(r+p)b + \gamma_3(m+b)p + \gamma_4(m+b)i$$
$$0 \leq \gamma_1,\gamma_2,\gamma_3,\gamma_4 \leq 1 \tag{5.11}$$

monetary relations

$$m = m_d(y,r+p,b) \qquad m_y,m_b>0;\ m_r<0 \tag{5.12}$$

$$r_e = r + p - p_e \tag{5.13}$$

budget constraints

$$z_1 = m + a + b \tag{5.14}$$

$$z_2 = 1-a \tag{5.15}$$

$$Da = -y_2 + i + C_2 - i.a \tag{5.16}$$

$$Db + Dm = -y_g + g - i.(b+m) \tag{5.17}$$

Equations 5.1 and 5.2 give the production function and the distribution of production between wages and profits. As in chapter 2 the income of each class is defined including interest payments net of inflation losses (eq. 5.3 and 5.4). In addition the present model also allows for unemployment benefits in worker's income. Equation (5.5) defines the income of the government sector as tax receipts less transfers and real interest on debt and money.

Consumption of each class is related to disposable income, wealth and the interest rate after taxes (eq. 5.7 and 5.8). It may be noted that consumption thus depends in two (possibly opposite) ways on the rate of interest: the actual return on assets (r) influences consumption through the amount of interest income, whereas the prospective yield of savings (r_e) determines the allocation of income between saving and consumption.[2] Taxes are levied on wages and transfer income (tax rate τ_0), real interest income of workers (τ_1) and net income of the corporate class (τ_2).[3]

Equation (5.11) describes the budgetary regime of the government. This relation is discussed in detail in the next section. Equation (5.12) defines portfolio equilibrium with a conventional money demand function (m_d). The prospective interest rate r_e (eq. 5.13), which is relevant to investment and saving decisions, may differ from the actual interest rate as a result of differences between the actual and the expected rate of inflation p-p_e. Finally, equations 5.14 to 5.17 define the wealth of both classes and give the budget constraints for the corporate sector and the government sector.

5.3 FISCAL POLICY REGIME

In the extensive literature on the dynamics of public debt, government behaviour is modelled in many different ways. One might distinguish two lines of thinking: the normative approach which investigates the economic consequences of particular budgetary regimes or rules, and the behavioural approach which attempts to give a positive explanation of fiscal behaviour. In most theoretical modelling both approaches lead, however, to some reduced form representation of government policy. We shall therefore not elaborate on this distinction, and follow common practice in representing government behaviour by a budgetary regime or rule.

The budgetary regime can be formulated in terms of the instruments (expenditure and tax rates) (cf. Blinder and Solow 1973, Christ 1968, 1978, 1979, Tobin and Buiter 1976) or in terms of some target such as the budget deficit (cf. Domar 1957) or the size of public debt (cf. Barro 1979). Moreover, in recent theoretical models more dynamic ('feedback') rules have been proposed for fiscal policy, relating taxes or expenditure to the evolution of macroeconomic variables such as unemployment or the ratio of public debt to national income (cf. Buiter 1986, Van de Klundert and Van der Ploeg 1987).

As our aim is to examine the consequences of different fiscal policy regimes for medium-period and long-period dynamics, we shall adopt a general representation of fiscal policy that encompasses each of the following regimes which are well-known in the literature:

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[2] By relating savings to the real interest rate it is assumed that there is no 'inflation illusion' or other inflationary distortions, for example arising from the tax system or liquidity constraints (due to the 'front-loading' effect of higher nominal interest rates).

[3] By attaching tax rates to real interest income, we implicitly assume that inflation losses on nominal assets are taxed at the same rate as nominal interest income. Although this is generally not true in practice, it seems a reasonable simplification in the context of the present analysis. Moreover, there is some evidence that positive and negative biases in the tax system more or less offset one another (cf. Tanzi 1984). For an excellent discussion on the consequences of inflationary biases in the tax system, see Feldstein (1983).

a. fixed tax rates and expenditure (Blinder, Solow 1973);
b. fixed tax rates and a fixed sum of expenditure and nominal interest payments (Christ 1979);
c. fixed tax rates and a fixed sum of expenditure and interest payments net of taxes (Tobin, Buiter 1976);
d. fixed budget deficit (Domar 1957, ch.II);
e. balanced budget (Buchanan 1978);
f. constant debt ratio (Barro 1979)

In terms of the model developed above, these regimes (named after the authors mentioned) may be written as:[4]

a. Blinder & Solow	g = constant
b. Christ	$g + (r+p)b$ = constant
c. Tobin & Buiter	$g + (1-\tau_1)(r+p)b$ = constant
d. Domar	$g - T_1 - T_2 + \tau_3 u + (r+p)b$ = constant
e. Buchanan	$g - T_1 - T_2 + \tau_3 u + (r+p)b = 0$
f. Barro	$g - T_1 - T_2 + \tau_3 u + (r+p)b = (i+p)(b+m)$

where $(r+p)$ is the nominal interest rate and $(i+p)$ the nominal growth rate of the capital stock. In the regimes a. to c. the tax rates are fixed as well.

The first regime is the standard textbook case with exogenous tax rates and government expenditure. As total outlays consist of interest payments as well as expenditure the budget deficit in this regime varies with debt service. Therefore, Tobin and Buiter (1976) suggested that the sum of expenditure and interest payments might be a better measure of the stance of fiscal policy than expenditure alone. This approach is followed by Christ, but he uses the sum of expenditure and *gross* interest payments while Tobin and Buiter take interest payments net of taxes levied on interest income. It may be noted that both regimes imply some form of *internal crowding out* as government expenditure has to be reduced automatically when interest outlays grow.

The last three regimes also imply some form of internal crowding out as they require government expenditure or taxes to be adjusted in order to realize the target for the budget deficit (regime d and e) or debt (regime f). The case of a constant budget deficit was originally investigated by Domar, who found on the basis of a partial analysis that a positive nominal growth rate $(i+p)$ was sufficient for stability

[4] We have followed the authors in their interpretation of the government deficit. There exist, however, some intricate difficulties with respect to the measurement of the budget deficit. See Appendix 5.A for a discussion of these difficulties.

of the accumulation of public debt. The 'classical' balanced budget regime, which is in fact a special - zero deficit - case of the Domar regime, is obviously the most restrictive of the regimes considered so far. This regime seems to have regained some new advocacy recently (cf. Buchanan et al. 1978). Note that according to this regime public debt should be *negative* in the long run. This follows from the budget constraint (5.17) and the fact that money stock cannot be negative: in the case of a zero budget deficit the steady state conditions $Db=Dm=0$ can only be satisfied if $(m+b)(i+p)=0$; hence $b<0$ whenever the stock of base money $m>0$ and $(i+p)\neq 0$.

According to the final regime, the government adopts a target directly for the size of public debt. This regime is suggested by Barro (1979) who also investigated the proposition that in reality governments aim at a constant ratio between public debt and national income.[5]

Now consider our representation of fiscal policy. Equation 5.11 gives a general formulation of fiscal policy, which relates government expenditure to tax receipts (if $\gamma_1>0$), interest outlays (if $\gamma_2>0$), inflationary erosion of liabilities (if $\gamma_3>0$) and the 'real' erosion as a result of income growth (if $\gamma_4>0$). By varying the γ coefficients this fiscal policy function can represent all regimes discussed above, and, of course, many kinds of hybrid regimes as well. The above summing up of regimes can be reduced to the following parameter settings:

regime	g_o	γ_1	γ_2	γ_3	γ_4
a. Blinder & Solow	g_o	0	0	0	0
b. Christ	g_o	0	1	0	0
c. Tobin & Buiter	g_o	0	$1-\tau_1$	0	0
d. Domar	g_o	1	1	0	0
e. Buchanan	0	1	1	0	0
f. Barro	0	1	1	1	1

5.4 THE MEDIUM PERIOD MODEL

The model considered so far leaves several loose ends. Nothing has been said about the monetary regime, nor about the determination of prices and income distribution. As the medium period is characterized by sluggish prices and disequilibrium between aggregate demand and aggregate supply, we shall assume that the decisive determinant

[5] See also Kremers (1986a, 1986b) for a criticism of Barro and an alternative (macroeconomic) modelling of government behaviour. For a more political explanation of government behaviour cf. Renaud, Van Winden (1987).

of investment is the aim to adjust capacity to the expected demand for goods.[6] This gives rise to a Harrodian modelling of the medium period. In contrast with Harrod however, we shall take account of the impact of the fiscal and the monetary regime on these dynamics.

In order to keep the model within manageable proportions we assume that the medium period is characterized by fixed technique of production and fixed profit share (π/y). For the moment we shall also neglect the accumulation of corporate and public debt. This can be motivated by the fact that public and corporate debt are slow variables in relation to the medium-term dynamics of aggregate demand and investment. The analysis of the accumulation of public and corporate debt is postponed till the long-period analysis in the next chapter. This does not mean, however, that the distribution of wealth is neglected in the present analysis; on the contrary, it will be seen below that the size of public and corporate debt has an important impact on the stability of the system in the medium period.

The essential element in the dynamics of this model concerns the interaction between investment and aggregate demand. In order to concentrate on the basic relationships, we neglect financial factors and postulate the following simple Harrodian investment function where the desired rate of investment i^* depends on the growth of demand ($\hat{y}_e$)[7] and the utilization rate ($h=y/y_c$). In accordance with Harrod it is further assumed that investment adjusts only gradually to the desired level[8]:

$$i^* = i^*(\hat{y}_e, h) \qquad i^*_y, i^*_h > 0 \; ; \; i^*(h=1)=\hat{y}_e \; ; \; i^*(h=0)=-\infty \tag{5.18}$$

$$Di = \vartheta_2(i^* - i) \tag{5.19}$$

The desired rate of investment i^* is positively associated with the growth of demand and the utilization rate. If capacity is fully utilized i^*, and thus the desired growth of capacity, is equal to the expected growth of demand. If, in the limit, utilization falls

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[6] See Kuipers (1981) and Van Ewijk (1982) for a discussion of the microeconomic foundation of this proposition.

[7] $\hat{y}_e = (DY/Y)_e$, where Y stands for real production or demand in absolute terms.

[8] In a comment on Kuipers (1981) we analysed a similar model. In contrast with the present analysis disequilibrium between capacity growth and demand growth in that model was caused by lagged expectations on demand growth rather than by slow adjustment of capacity to the desired level as in the present model (Van Ewijk 1982b). It can easily be seen that this alternative approach eventually produces exactly the same equation for Di. Assume the following adaptive expectations function:

$$D\hat{y}^e = \vartheta_3\{\hat{y}-\hat{y}^e+\vartheta_1(1-1/h)\}$$

where $\hat{y}^e$ stands for the growth of expected demand (DY_e/Y_e) which must be distinguished from the expected growth of demand $(DY/Y)_e$ (Van Ewijk 1982b, p.108). Now assuming that capacity is at every instant fully adjusted to expected demand, it is found that $i=i^*=\hat{y}^e$. Substitution in the adaptive expectations function also yields eq. 5.23. A difficulty with respect to this approach is, however, that the modelling of expectations does not satisfy the weak consistency axiom (as it allows for discrepancy between y and y_e), while the present approach does (Turnovsky and Burmeister 1977).

to zero desired (h=0) investment tends to $-\infty$. This is obvious because at zero demand for their products firms want to close down all existing capacity. In the following analysis we shall adopt the following explicit function which satisfies these characteristics

$$i^* = \hat{y}_e + \vartheta_1(1-1/h) \tag{5.18'}$$

In order to ease our analysis, we shall further use a linear version of the consumption functions (5.7 and 5.8), thus

$$C_j = c_j y_j + c_{jr}(1-\tau_j)r_e + c_{jz}z_j + c_{jo} \qquad \text{for } j=1,2 \tag{5.20}$$

where $0<c_j<1$, $c_{jr}<0$ and $c_{jz}>0$.

Statics

Before analysing the dynamics we shall look in more detail at the static solution of the model. After substitution for C_1, C_2 and g we obtain for aggregate demand y_d

$$y_d = c_o + c_y y + c_r r + c_{re} r_e + c_{1z} m + \{c_p m - (\gamma_2 - \gamma_3)b)\}p + \{1+\gamma_4(m+b)\}i \tag{5.21}$$

where

$$c_o = c_{1o} + c_{2o} + g_o + c_{1z}(a+b) + c_{2z}(1-a) + (c_1-\gamma_1)(1-\tau_0)\tau_3 l_s$$

$$c_y = c_1(1-\tau_0)(1-\pi/y) + c_2(1-\tau_2)\pi/y - (1-c_1-\gamma_1)(1-\tau_0)\tau_3 l_c/y_c + c_1\{\tau_0(1-\pi/y)+\tau_2\pi/y\}$$

$$c_r = c_1(1-\tau_1)b + \{c_{1(}1-\tau_1)-c_2(1-\tau 2)\}a - \gamma_2 b + \gamma_1\{\tau_1 b+(\tau_1-\tau_2)a\}$$

$$c_{re} = c_{1r}(1-\tau_1) + c_{2r}(1-\tau_2)$$

$$c_p = -c_1(1-\tau_1) - \gamma_1\tau_1 + \gamma_3$$

Equation (5.21) brings out that aggregate demand is positively associated with autonomous expenditure c_o, investment i and income y ($c_y>0$), and negatively with the prospective interest rate ($c_{re}<0$). The impact of the actual interest rate (c_r) is, however, uncertain: it depends on the distribution of wealth among the three sectors of the economy, and on the fiscal reaction to changes in interest outlays and tax receipts. As normally a>0, a higher interest rate generally leads to a redistribution of income in favour of workers, which causes aggregate consumption to *rise*. Further, if b>0, the higher interest rate also leads to a redistribution of income from the government to workers. The impact of this income shift on aggregate expenditure depends on the budgetary regime: if government expenditure and taxes are autonomous (γ_2=0), as in the Blinder-Solow regime, aggregate expenditure will rise as workers spend more ($c_r>0$). However, if $\gamma_2>0$ as in all other regimes, the higher consumption of workers may be compensated by lower expenditure by the government; hence aggregate expenditure may fall ($c_r<0$).

Note that if $c_r>0$ and $c_{re}<0$ the overall impact of the interest rate is uncertain. If the distributive effects are strong relative to the 'substitution' effect of a higher prospective interest rate (c_{re}), a higher interest rate might well lead to a *larger* aggregate demand. It can be established from the above aggregate demand function that the occurrence of such a *reverse interest effect*[9] becomes more likely if (see 5.21)

1. the debt of the government and the corporate sector is larger,
2. the marginal propensity to consume of workers is higher,
3. the government reponds less to changes in debt service.

The impact of inflation (p) on demand is given by

$$\frac{dy_d}{dp} = \{c_p m - (\gamma_2 - \gamma_3)b\} + \{c_r \frac{dr}{dp} + c_{re} \frac{dr_e}{dp}\} \gtrless 0$$

Inflation leads to a redistribution between the government and the workers through the erosion of real balances. Just as in the case of the interest rate, the overall effect of inflation on spending (dy_d/dp) thus depends on these sectors' reactions to the change in income (the first term in the equation above). As far as inflation also affects the real interest rate on public and corporate debt the ensuing redistribution will affect spending as well (the second term). The overall effect may thus again be negative as well as positive. The occurrence of a positive effect of prices on demand, or a *reverse Pigou effect* (cf. Tobin 1980), is more likely as γ_3 is higher; that is if the government takes better account of the erosion of its liabilities through inflation. This is particularly the case if it adopts some target for its debt, as in the Barro regime.

A simple Harrodian model

If we follow Harrod and neglect any feedback from the monetary sector, this model can be shown to exhibit the well-known knife-edge dynamics. Thus assuming the interest rate and the inflation rate to be fixed, and neglecting the impact of the real money stock on demand, aggregate demand becomes a simple linear function of income and investment

$$y_d = y_o + c_y y + \{1+\gamma_4(m+b)\}i \tag{5.22}$$

where c_y is given above and y_o comprises all other factors. Since for a constant capital output ratio

[9] In analogy with the discussion on the Pigou effect (cf. Tobin 1980), one might call this positive interest effect a reverse Keynes effect.

$$\hat{y} = i + Dh/h$$

Noting that $h=y/y_c$, and assuming $y=y_d$ and naive expectations ($\hat{y}_e = \hat{y}$), we obtain after some manipulation

$$Di= \vartheta_1\vartheta_2 \frac{i+y_o-(1-c_y)y_c}{i+y_o-\vartheta_2} \tag{5.24}$$

Provided that the speed of adjustment of investment ϑ_2 is not too large (so that $\vartheta_2<(1-c_y)y_c$)[10], this result entails a positive relation between Di and i and therefore the well-known knife-edge characteristics of the warranted rate of growth.[11]

This model hinges, however, on strong assumptions about the monetary sector. In fact, it totally neglects the role of money which may be expected to have a dampening impact on the disequilibrium dynamics. Therefore we shall now consider a more sophisticated model which takes account of the feedback from the monetary sector.

5.5 MONETARY FEEDBACK

Now let price formation and money growth be represented by the following relations:

$$p = \vartheta_3(h-1) + p_o \qquad \vartheta_3>p_o{}^{12} \tag{5.25}$$

$$p_e = p_o + \vartheta_4(p-p_o) \tag{5.26}$$

$$Dm= m(\hat{m} - i - p) \tag{5.27}$$

where $p_e=(DP/P)_e$ represents the expected rate of price change and $\hat{m}$ the nominal growth of money stock (in absolute terms). Equation 5.25 relates inflation to the rate of utilization h and core inflation p_o.[13] Note that the modelling of inflation and investment is 'Keynesian' in the sense that for low utilization rates investment adjusts

[10] If $\vartheta_2>(1-c_y)y_c$ it can be seen that - evaluated in the equilibrium (Di=0) - the second order effect of Di exceeds the initial effect, so that the sign of the total effect is reversed. Although the differential equation becomes stable now, nevertheless the solution can therefore not be regarded as a stable equilibrium (see also Van Ewijk 1982b).

[11] The warranted growth rate is given by $i=(1-c_y)y_c-y_o$ where Di=0. Remembering that i is equal to the growth rate of production and that y_c is the reciprocal of the capital coefficient the similarity between this result and Harrod's is obvious.

[12] This condition ensures that inflation becomes negative for low utilization rates.

[13] This function is akin to that of Christ (1979), but instead of his exogenous expected rate of inflation p_e we have included core inflation p_o in this function and introduced a distinct function for expected inflation depending on core inflation and actual inflation.

faster than inflation ($i \rightarrow -\infty$ but $p \rightarrow (p_o - \vartheta_3)$ if $h \downarrow 0$) while at high utilization rates inflation reacts relatively sharper than investment. The modelling of inflation expectations (5.26) implies a partial adjustment of expected inflation to actual inflation. This equation satisfies the weak consistency axiom, as there is no difference between the expected and the actual level of prices at the present moment. However, as we do not assume perfect myopic foresight with respect to the *change* in prices ($p_e \neq p$), the strong consistency axiom is, of course, not satisfied (Burmeister and Turnovsky 1976). Further, note that these equations together yield a consistent medium-term equilibrium at $h=1$ characterized by $p=p_o=p_e$. Equation 5.27 for the change in money supply (in relation to capital stock) follows simply from differentiation of m with respect to time.

As for the moment we neglect changes in nominal public debt, we may for convenience choose the following function for the money demand

$$m_d = \mu y/(r+p-r_o)$$

which implies in case of portfolio equilibrium ($m=m_d$)

$$r = \mu(y/m) - p + r_o \tag{5.28}$$

This function has the attractive properties that $\lim_{m \to 0} r = \infty$ and $\lim_{m \to \infty} r = r_o - p$. The first characteristic implies that the interest rate can in principle be pushed up infinitely by monetary contraction; the second characteristic represents the liquidity trap.

After substitution for p, r and r_e (eqs. 5.25, 5.26, 5.28) in aggregate demand (5.21), the utilization rate can be solved in terms of i and m:

$$h = m. \frac{\{1+\gamma_4(m+b)\}i + h_1 m + h_2}{-c_p \vartheta_3 m^2 + h_3 m - (c_r + c_{re})\mu y_c} \tag{5.29}$$

where

$$h_1 = c_{1z} - c_p(\vartheta_3 - p_o)$$

$$h_2 = c_o + c_r(r_o - p_o + \vartheta_3) + c_{re}(r_o - p_o + \vartheta_3\vartheta_4) + (\gamma_2 - \gamma_3)(\vartheta_3 - p_o)b$$

$$h_3 = (1-c_y)y_c + (c_r + \vartheta_4 c_{re})\vartheta_3 + (\gamma_2 - \gamma_3)b\vartheta_3$$

For economic reasons it is assumed that $h>0$ and that the numerator must be positive. This latter requirement represents the conventional assumption that the impact of y on aggregate demand should be less than unity. Hence

$$-c_p \vartheta_3 m^2 + h_3 m - (c_r + c_{re})y_c > 0 \tag{5.30}$$

Given this condition it can be seen that the utilization rate is a positive function of the rate of investment provided that $(1+\gamma_4(m+b)>0)$, thus as long as the government does not overcompensate the change in i by an opposite change in its expenditure g. As this can occur only if government liabilities are very negative $(m+b<0)$, we shall leave this exceptional case out of consideration. Also money supply normally has a positive impact on the utilization rate $(h_m>0)$. However, in the case of a reverse interest effect $(c_r+c_{re}>0)$ a rise in money supply, and thus a fall in interest rate, might well lead to *lower* utilization of capacity.

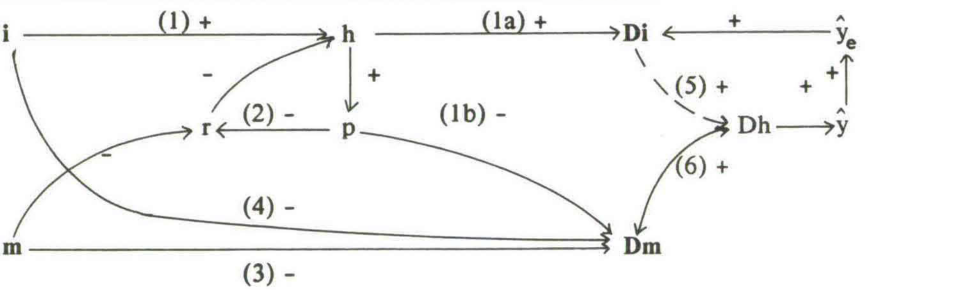
Dynamics

After substitution for i* (5.18) in the Di function (5.19) and for p in the equation for Dm (5.27) the model yields the differential system

$$Di = \vartheta_2(\hat{y}_e - i) + \vartheta_2\vartheta_1(1-1/h) \qquad (5.31)$$

$$Dm = m(\hat{m} - i - p_o - \vartheta_3(h-1))$$

where h is the known function of i and m (5.29). First, note that the steady state solution of the system (Dm=Di=0) is characterized by full capacity utilization (h=1), core inflation $(p=p_o)$ and a rate of investment equal to the growth of real money stock $(i=\hat{m}-p_o)$. As to the dynamics, the following scheme illustrates the essential causative links (the scheme is made up for the case with a normal interest effect on demand):



The fundamental destabilizing causation arises from the positive linkage of Di to i through the utilization rate (linkage 1a in the scheme). This is the typical Harrodian causation from higher investment to larger demand and thereby to a further stimulation of investment. In the present model the effect of utilization h on demand is magnified by the induced rise in inflation p and the consequential fall in the real interest rate r (linkage 2). The monetary sector seems to have a principally stabilizing influence on the system, on the one hand through the negative linkage of Dm to m

(3), but more importantly, through the negative feedback of a higher growth rate i and higher inflation p on the growth of money stock Dm (linkages 4 and 1b).

The dashed lines give the additional effects arising from the impact of Di on the growth of (expected) demand, and thereby on investment again. On the one hand this may reinforce the destabilizing impact of investment through the positive feedback of higher investment growth on demand growth (linkage 5); on the other hand it also exerts a stabilizing influence as it makes the impact of money on investment stronger (linkage 6).

The phase diagram

As a first approximation of the dynamics of this model, we shall concentrate on the role of the utilization rate, and thus neglect for the moment the influence of expected demand growth on investment (linkage 5 in the scheme). In terms of the model we therefore set $\vartheta_2\vartheta_1=\vartheta$ (>0) and $\vartheta_2=0$. The dynamics of this system can be seen from the phase diagram (*figure 5.1*) which depicts the Di=0 and the Dm=0 conditions in the {i,m} plane. In order to bring out the essential features of the system the {i,m} plane is projected on an infinite horizon (cf. Jordan, Smith 1987). As m>0 only the positive hemisphere needs to be considered.

In order to ease our argument we restrict our analysis to the case with $c_p=0$, which means that we neglect the impact of inflationary erosion on consumption. Together with the plausible assumption that $h_3>0$ this allows us to neglect the constraint of a positive denominator of h (see 5.30). The conditions Di=0 and Dm=0 conditions can then be written as the following functions F and G for i:

Di=0 if i=F(m) where

$$F(m)= \frac{-c_{1z}m - (c_r+c_{re})(\mu y_c/m+r_o-p_o) - c_o + (1-c_y)y_c + (\gamma_2-\gamma_3)p_o b}{1 + \gamma_4(m+b)} \tag{5.32}$$

Dm=0 if i=G(m) where

$$G(m) = \vartheta_3 \frac{c_{1z}m^2 + j_1 m + j_2}{h_3 m - (c_r+c_{re})\mu y_c + \vartheta_3\{1+\gamma_4(m+b)\}m} \tag{5.33}$$

$$\begin{aligned} \text{where } j_1 &= -c_o - (c_r+c_{re})r_o + \hat{m}(c_r+\vartheta_4 c_{re}) + (\gamma_2-\gamma_3)\hat{m}b + \\ &\quad + (\hat{m}-p_o+\vartheta_3)(1-c_y)y_c/\vartheta_3 + c_{re}(1-\vartheta_4)p_o \\ j_2 &= -(\hat{m}-p_o+\vartheta_3)(c_r+c_{re})\mu y_c/\vartheta_3 \end{aligned}$$

and h_3 is given above (eq. 5.29). Note that the denominator of G(m) is equal to the denominator of the utilization rate h except for the last term. Since this term is always ≥ 0 we can conclude that the denominator of G is positive for all m as well.

First, concentrating on the case of a 'normal' interest effect ($c_r+c_{re}<0$) we are able to establish the following characteristics of these functions (F_m and G_m denoting the first derivatives with respect to m):

	$F(m) \gtrless 0;$	$F_m(m) < 0$	for all m
	$F(0) = \infty;$	$F_m(0) = -\infty$	
	$F(\infty) = -c_{1z}/\gamma_4;\ (<0)$	$F_m(\infty) = 0$	if $\gamma_4>0$
or	$F(\infty) = -\infty;$	$F_m(\infty) = -c_{1z}\ (<0)$	if $\gamma_4=0$
	$G(m) \gtrless 0;$	$G_m(m) \gtrless 0$	for all m
	$G(0) = \hat{m}-p_o+\vartheta_3\ (>0);$	$G_m(0) \gtrless 0$	
	$G(\infty) = -c_{1z}/\gamma_4\ (<0);$	$G_m(\infty) = 0$	if $\gamma_4>0$
or	$G(\infty) = -\infty$	$G_m(\infty) = -c_{1z}\ (<0)$	if $\gamma_4=0$

Since h is positively related to investment it can further be seen from (5.31) that

$$Di > 0 \iff i > F(m) \quad \text{and} \quad Dm < 0 \iff i > G(m)$$

The shape of the Di=0 curve in *figure 5.1* follows directly from the above characteristics. Starting from the upper left corner $\{\infty,0\}$ this curve slopes gradually downward. For the Barro regime where $\gamma_4>0$ (or if $c_{1z}=0$) it reaches the horizon at the m-axis as the F-curve becomes parallel to this axis at infinity. For all other regimes (where $\gamma_4=0$) the Di=0 curve reaches the horizon below the m-axis. The shape of the Dm=0 curve is less straightforward, but it is at least certain that it starts at a finite value on the i-axis and reaches the horizon at the same point as the F-curve.

Further, it can be established that, for the case with a normal interest effect, the steady state condition h=1 always yields one positive and one negative solution for m.[14] Hence we can conclude that there always exists a *unique steady state solution* in

[14] For h=1 one obtains the following quadratic expression for m

$$(c_{1z}+\gamma_4 i)m^2 + \{h1-h_2+(1+\gamma_4 b)i\}_m + (c_r+c_{re})\mu y_c = 0$$

Excluding a negative steady state growth rate (hence $i \geq 0$) this equation always yields one positive and one negative solution for m when the interest effect on expenditure is negative ($c_r+c_{re}<0$). If the interest effect is reversed this equation will yield either two positive solutions or two negative solutions, or no solution at all. This case will be considered in the next section.

the relevant hemisphere ($m>0$).

There is yet another condition which must be taken into account, namely the condition of a non-negative utilization rate ($h \geq 0$). This condition is satisfied if (see eq. 5.29)

$$\{1+\gamma_4(m+b)\}i + c_{1z}m + h_2 > 0 \tag{5.34}$$

It can easily be seen that this lower boundary has a negative slope in the $\{i,m\}$ plane. In the limit, when $m \to \infty$ the slope becomes equal to 0 (or $-c_{1z}$ if $\gamma_4=0$); this implies that this curve reaches the horizon at the same point as the $Di=0$ and $Dm=0$ curves. Further, it can be seen that the $Di=0$ curve lies above this boundary for any $m>0$. If i approximates this lower boundary curve the utilization rate becomes zero, so that the desired rate of investment and hence Di tend to $-\infty$. This $h=0$ boundary thus operates as a sort of 'black hole' which absorbs all phase paths that come too close to it.

On the basis of these global results we can make several inferences. Intersection S of these curves is an equilibrium point which may or may not be stable. If it is stable it can, however, only be *locally* stable; there always exists some area around the $h=0$ curve from where the system can never return to S. Thus S is at best stable within a certain zone. This means that after small shocks the system will return to S, but that large disturbances may push the system beyond the critical 'stable' zone, so that the system gets unbalanced for ever. As the point at $\{\infty,0\}$ can be shown not to be stable it can be inferred that the system cannot move upward for ever.[15] Therefore it can be concluded that any path outside the stable zone must sooner or later come into the 'attraction zone' of the $h=0$ curve.

These results indicate that the introduction of monetary feedback has a stabilizing impact on the Harrodian model; a stable equilibrium (the warranted rate of growth)

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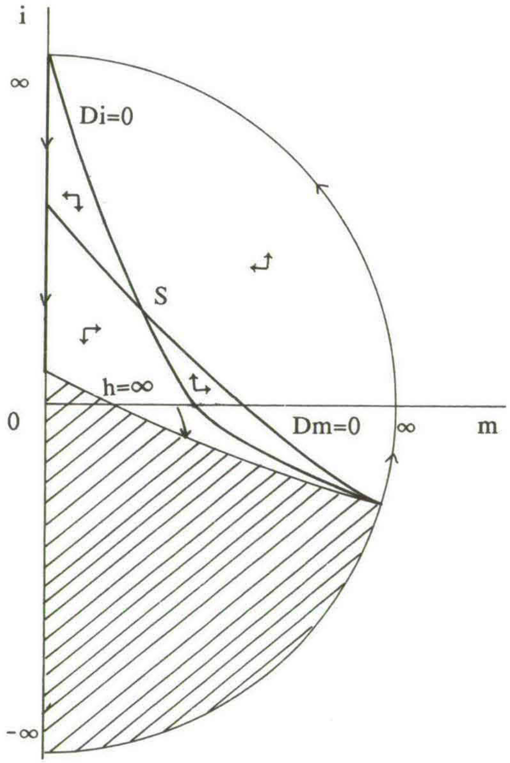
[15] By expressing the model in polar coordinates we get the following equation for the radius r:

$$r.Dr = i\vartheta(1-1/h)+m^2\{\hat{m}-i-p_o-\vartheta_3(h-1)\}$$

After substitution for h it can be seen that for a sufficiently large r (and thus large m and i) the sign of Dr is determined by the sign of the term $-\vartheta_3\gamma_4 m^3 i/h_3$ (or $-\vartheta_3 m^2(i+h_1 m)/h_3$ if $\gamma_4=0$) Since $h_1,h_3>0$ this implies that there exists a finite circle around the origin where the sign of Dr is negative for any positive i and m; that is, at a sufficiently wide circle around the origin every path in the first quadrant crosses this circle bending towards the origin. Therefore we can conclude that a path tending towards $\{\infty,0\}$, and thus moving upward for ever, cannot exist.

Figure 5.1 Phase diagram



may exist, but only within certain boundaries. Although the monetary feedback appears to provide a sufficient check in an upward direction, there still always exists some zone along the h=0 boundary where the system gets destabilized; starting from a point in this zone the system will fall into a Harrodian process of accelerating decline in demand and reduction of investment. Note that these overall dynamics correspond remarkably well to what Leijonhufvud (1969) called the '*corridor*' characteristic which in his view is typical of Keynesian dynamics.

Local stability

Now let us look somewhat closer at the stability of equilibrium point S. Therefore consider the linearization of the system (5.31) evaluated in the steady state $\{m_s, i_s\}$:

$$\begin{bmatrix} Dm \\ Di \end{bmatrix} = \begin{bmatrix} -m\vartheta_3 h_m & -m(1+\vartheta_3 h_i) \\ \vartheta h_m & \vartheta h_i \end{bmatrix} \cdot \begin{bmatrix} m-m_s \\ i-i_s \end{bmatrix} \tag{5.35}$$

where h_m and h_i stand for the first derivatives of h with respect to m and i. From this system we obtain the following Routh-Hurwitz conditions for stability

$$\begin{aligned} RH1 &= \vartheta h_i - m\vartheta_3 h_m < 0 \\ RH2 &= m\vartheta h_m > 0 \end{aligned} \tag{5.36}$$

Still confining our analysis to the case with a normal interest effect $(c_r+c_{re})<0$, so that always $h_m>0$[16], it can be seen that the first condition (RH1) is decisive for stability. Unfortunately RH1 yields no neat algebraic solution. Nevertheless, it is evident that the reaction speed of investment ϑ is a crucial determinant of the stability of S. This result is straightforward as the 'acceleration' factor is known to be the principal factor in the instability of Harrodian models. The first condition (RH1) brings out that a critical value of this parameter exists beyond which the system becomes unstable. Denoting this critical value by ϑ_{max} we find

$$\vartheta_{max} = \vartheta_3 \frac{h_m m}{h_i} \tag{5.37}$$

For any $\vartheta<\vartheta_{max}$ the equilibrium is (locally) stable, and for any $\vartheta>\vartheta_{max}$ it is unstable. In other words, whenever prices show some flexibility ($\vartheta_3>0$) the equilibrium may be stable if ϑ is sufficiently low, that is, if investment does not react too sharply to deviations in the utilization rate.[17]

Hopf bifurcation

Further characteristics of equilibrium point S can be derived from *figure 5.2*. This figure shows the Routh-Hurwitz conditions in relation to ϑ. This figure brings out that S is a stable node for low values of ϑ; as ϑ becomes larger (but still $<\vartheta_{max}$) the

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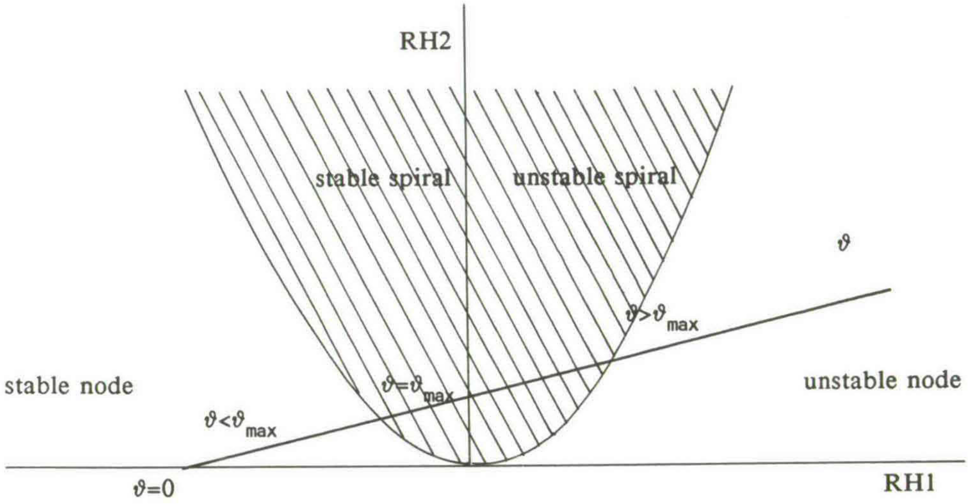
[16] The exact condition for $h_m>0$ evaluated in h=1 is $c_{1z}-(c_r+c_{re})\mu y_c/m^2+\gamma_4 i>0$. For any $i_s>0$ this condition implies that $h_m>0$ whenever $c_r+c_{re}<0$. Only in the case of a strong reverse effect of the interest rate on consumption might h_m be negative.

[17] Note that the steady state solution is independent of ϑ (eq. 5.28). It does however depend on ϑ_3.

Figure 5.2 Characteristics of equilibrium S



equilibrium becomes a stable spiral. If ϑ rises above ϑ_{max} S becomes an unstable spiral and eventually an unstable node.

The critical case with $\vartheta=\vartheta_{max}$ may represent a centre or spiral. If it is a spiral the system is characterized by a *Hopf-bifurcation.* In that case it is known that a limit cycle will exist around the equilibrium point S, the radius of which will vary with ϑ (Jordan and Smith 1987, 327-329). Unfortunately the system is too complex to determine the characteristics of this critical point algebraically. Therefore we shall proceed with some numerical simulations for plausible parameters of the model.

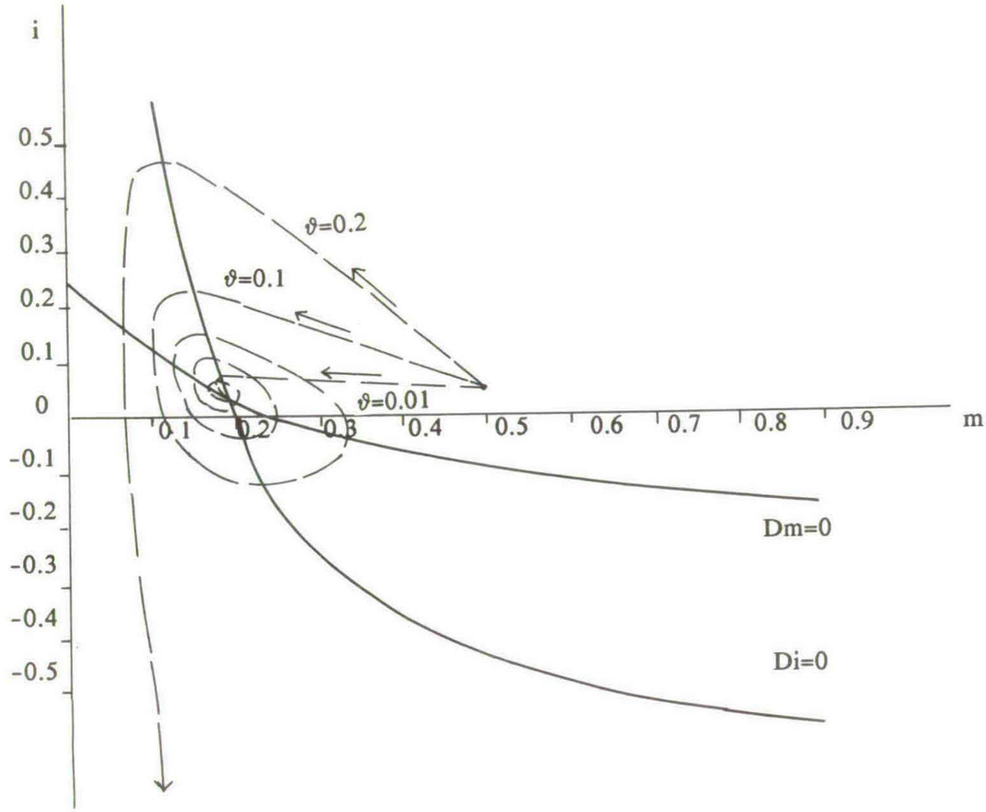
Unstable limit cycle

The above inferences about the characteristics of S are corroborated by *figure 5.3* which shows the simulation of the adjustment trajectories from a given starting point {0.05,0.5} for different values of ϑ (0.01; 0.1; 0.2). For the parameters on which these simulations are based the equilibrium S is characterized by:

i = 5%
m = 0.18
π = 8.2%
r = 3.9%
h = 1
p = 2%

As in this case ϑ_{max} = 0.151 the phase paths for ϑ=0.01 and ϑ=0.1 are stable (viz. a node and a spiral) while the phase path for ϑ=0.2 produces an unstable process, which eventually collapses onto the h=0 boundary.

Figure 5.3 Adjustment trajectories for ϑ = 0.01; 0.1; 0.2



Explanation: This figure is based on the following numerical values[18] r_o=0.02; p_o=0.02; g/y_c0=0.22; $\tau_0=\tau_1=\tau_2$=0.2; τ_3=0.3; b/y_c=0.5; a=0.5; c_1=0.9; c_2=0.6; c_{1r}=-1; c_{2r}=-1; c_{1z}=0.05; c_{2z}=0.05; c_{1o}=-0.015; c_{2o}=-0.01; μ =0.1; l_s=0.33; ϑ_3=0.2; ϑ_4=0.2; m=0.07; w=1; y_c=0.38; β=0.8; $\gamma_1,\gamma_2,\gamma_3,\gamma_4$=0. The simulations are based on an approximation of the differential system by the Runge-Kutta method (cf. Cohen 1973)

Further this numerical example can be seen to produce an *unstable limit cycle* around S when $\vartheta<\vartheta_{max}$. This cycle is unstable because whenever the system starts from a point inside this cycle it will eventually tend towards the equilibrium point S, but when it starts outside the cycle the trajectory will recede from the cycle for ever. The size of this cycle varies with ϑ. This implies that the system becomes more stable, i.e. the limit cycle becomes wider, as the reaction speed of investment is less. This is illustrated by *figure 5.4* which shows the limit cycles for three different values of ϑ

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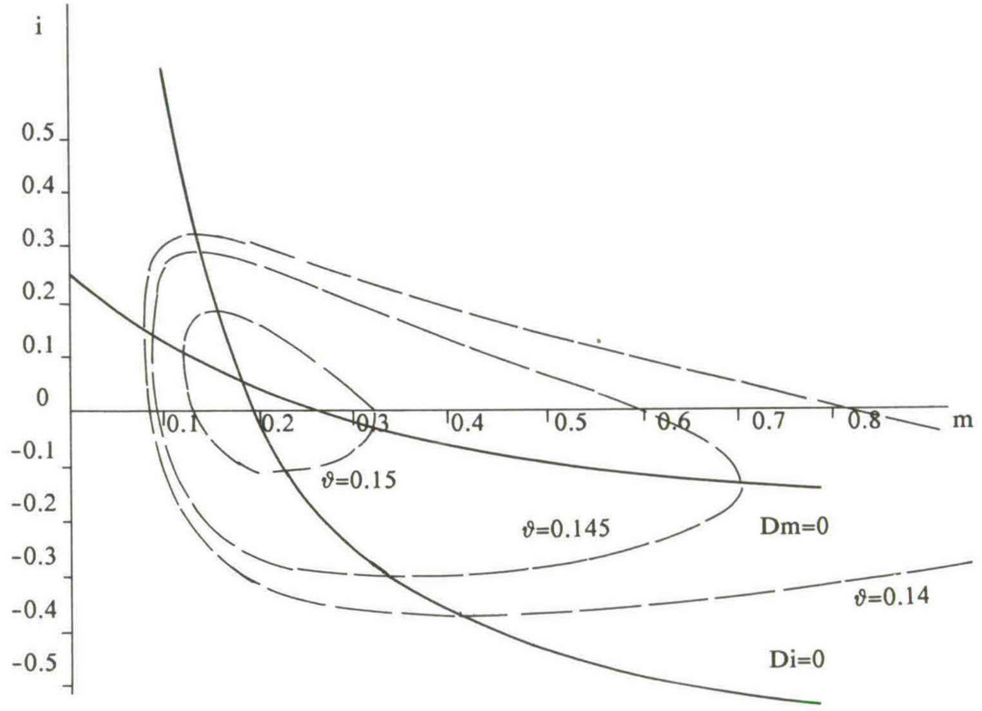
[18] In the next chapter we shall give a motivation for these parameter values.

Figure 5.4 Limit cycles for ϑ = 0.140; 0.145; 0.150



Explanation: see figure 5.3 above.

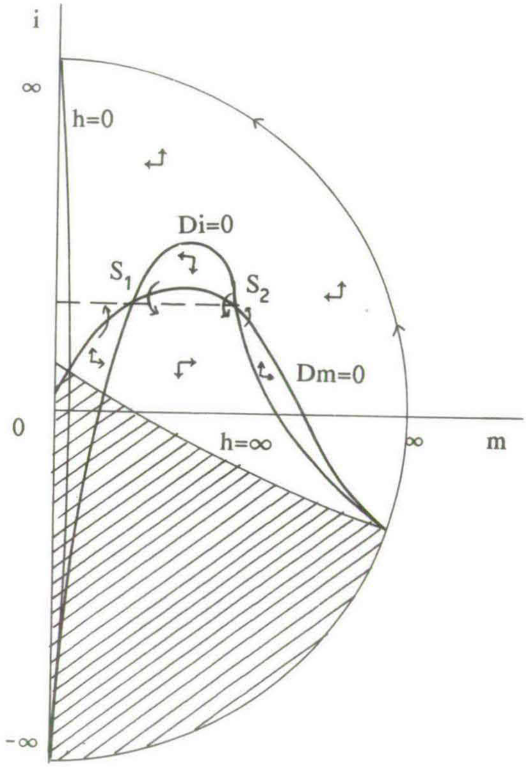
close to ϑ_{max} (0.140; 0.145; 0.150). For $\vartheta > \vartheta_{max}$ the limit cycle vanishes completely so that S becomes unstable globally as well as locally. It is evident that for ϑ's close to the critical value (for example ϑ=0.150), where equilibrium S is still stable locally, even a small displacement from S may be sufficient to push the system beyond the limit cycle and lead it away from S for ever.

5.6 SOME EXTENSIONS

Reverse interest effect

Figure 5.5 gives a phase diagram for the case with a reverse interest effect $(c_r+c_{re})>0$. As the interest effect becomes dominant for low m (when the interest rate is high) the Di=0 function approximates the y-axis now at - ∞. As a consequence the system has

Figure 5.5 Phase diagram with reverse interest effect



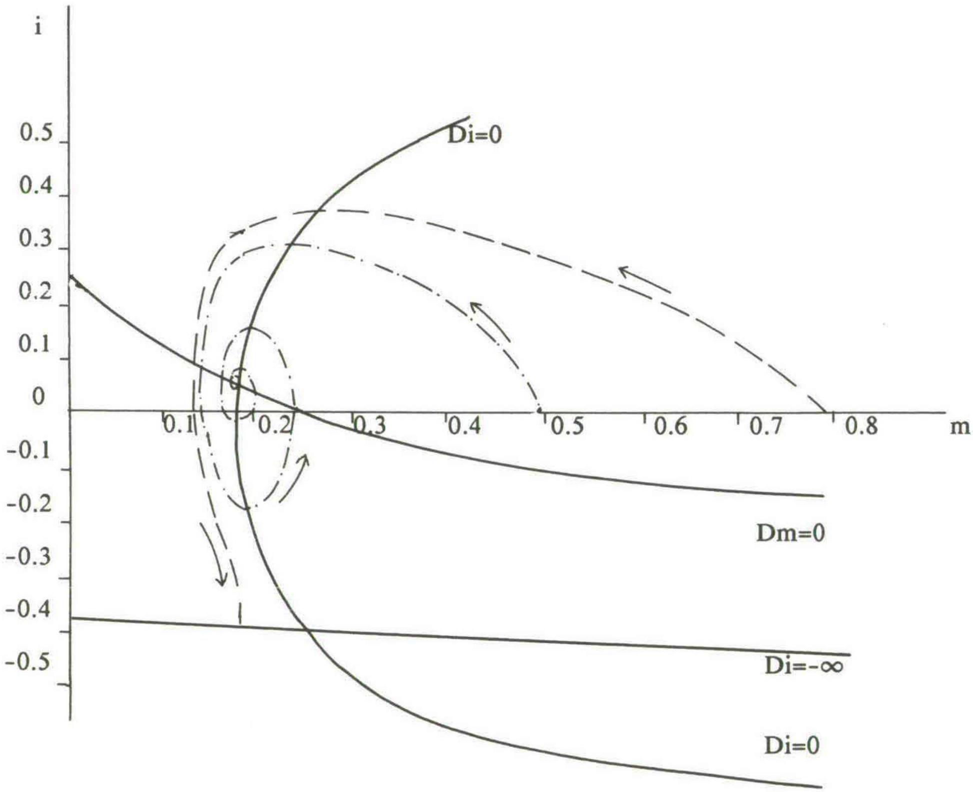
either two solutions or no solution at all in the positive hemisphere. The figure illustrates the case with two solutions (Sl and S2). Both solutions, of course, entail the same equilibrium for the growth rate ($i=\hat{m}-p_o$), but different solutions for the stock of money. The figure brings out that only one of these solutions may be stable (S2); the other solution (S_1) is evidently a saddle point, and thus unstable. Therefore we can conclude that if S2 is locally stable again, some zone must exist around S beyond which the system is unstable. If S2 is not stable, there is no stable finite solution at all.

In this case we should also take account of the condition that there be a positive denominator of h (eq.5.30), which implies a lower boundary for money stock ($m>(c_r+c_{re})y_c/h_3$). If the system approaches this boundary it can be seen that $h\rightarrow\infty$ and thus $Di/Dm\rightarrow0$. This means that the economy comes into an upward spiral of ever increasing demand and inflation. Hence we can conclude that, in contrast with the case of a normal interest effect, the system may be unstable in an upward as well as a downward direction.

Demand expectations

So far we have neglected the impact of demand expectations on investment (ϑ_2=0). *Figure 5.6* exhibits the phase diagram for the same numerical example, but now with partial adjustment of investment to expected demand growth (ϑ_2=0.5). For simplicity it is assumed that entrepreneurs have naive expectations with respect to demand growth ($\hat{y}_e=\hat{y}$). Although the steady state position is not changed, this figure brings out some remarkable changes. In the first place the Di=0 curve is no longer a monotonic decreasing function. Instead it bends forward in the region above the Dm=0 curve. This is due to the positive impact of Dm on demand growth and thereby on desired investment. A second remarkable difference is the appearance of a new

Figure 5.6 Phase diagram ϑ_2=0.5



lower boundary where $Di \rightarrow -\infty$. This boundary represents the condition that the secondary effect of demand growth on Di (linkage 5 in scheme 4.1) should not exceed the primary effect. This new lower boundary can always be shown to be more restrictive than the h=0 condition; hence this boundary lies above the h=0 curve for all m>0.[19]

Apart from these differences, the dynamics exhibits basically the same characteristics. At the given parameter values the equilibrium is stable, so that there exists a critical zone around this equilibrium. The figure shows two trajectories, one starting within the critical zone at {i=0;m=0.5) and the other starting outside it at {i=0;m=0.8}.

5.7 FISCAL POLICY AND STABILITY

In order to asses the impact of various (policy) variables on the stability of the system *table 5.1* gives the partial effects of these variables on ϑ_{max} (eq.5.37). As we have established above, this parameter has a clear cut impact on stability while it does not change the equilibrium position. If a variable has a positive effect on this critical value, ϑ_{max}, we may conclude that it has a stabilizing impact on the system.[20] A negative effect corresponds to a destabilizing impact. All effects are given in relation to the reference case given in *figure 5.4* above.

A general conclusion that emerges from these results is that all factors raising autonomous expenditure improve the stability of the system. Further, these results corroborate our earlier inference that the system becomes more stable as the monetary feedback is stronger: the sensitivity of the interest rate to money and income (μ) as well as the interest effects on consumption (c_{1r},c_{2r}) and the degree of price-flexibility (ϑ_3) have significant positive effects on ϑ_{max}. In addition, the system becomes more stable as money growth ($\hat{m}$) is faster, core inflation (p_o) higher and the autonomous interest rate (r_o) lower. At the given parameter values the effect of demand expectations on investment (ϑ_2) is to reduce stability. Also a larger corporate debt tends to destabilize the system, which is obvious as the ensuing distribution effect weakens the negative feedback of the interest rate on expenditure.

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[19] Since $\hat{y}_e = i + Dh/h = i + (h_i/h)Di + (h_m/h)Dm$ we can find from (5.18 and 5.19) that

$Di = \vartheta_2\{(h_i/h)Di + (h_m/h)Dm\} + \vartheta_1\vartheta_2(1-1/h)$.

Now requiring that the secondary effect $\vartheta_2(h_i/h) < 1$ we obtain the condition $i > \vartheta 2 - (h_1 m + h_2)/\{1+\gamma_4(m+b)\}$ whereas the condition for h>0 was $i > -(h_1 m + h_2)/\{1+\gamma_4(m+b)\}$. For any $\vartheta_2 > 0$ this boundary thus lies above the h=0 boundary for any m>0.

[20] This concerns only the local stability of the system. For an indication of the impact on the global stability one should obtain some measure of the change in the area within the limit cycle. As this entails a very laborious computational procedure we confined our analysis to the above measure of local stability.

Table 5.1 Partial effects on ϑ_{max}

g_o	0.076	c_1	-0.008
γ_1	-0.004	c_2	0.055
γ_2	0.021	c_{12}	-0.083
γ_3	-0.0002	c_{2r}	-0.083
γ_4	-0.028	c_{1z}	-0.164
τ_0	-0.055	c_{2z}	-0.099
τ_1	-0.039	c_{1o}	0.199
τ_2	-0.145	c_{2o}	0.199
τ_3	0.004	a	-0.025
μ	0.015	r_o	-0.266
$\hat{m}$	0.198	p_o	0.073
ϑ_2	-10.08	w	0.015
ϑ_3	0.754	l_s	0.043
b	-0.025	y_c	-0.084

Explanation: all effects are measured with reference to the steady state given in figure 5.4 above The effect of the γ coefficients have been corrected for the impact effect on government expenditure. One should be careful to compare the absolute effects because an equal change in each variable may produce quite different impacts in absolute amounts.

With regard to fiscal policy these results indicate that the system becomes more stable as autonomous government expenditure and unemployment benefits are higher and taxes are lower. The Christ regime, with γ_2=1 and the other γ coefficients equal to zero, clearly emerges as the best policy regime to choose for the medium period. This conclusion proves to be robust for different parameter sets in the neighbourhood of our reference set. This is not really surprising as the γ_1,γ_3 and γ_4 parameters relate expenditure to pro-cyclical variables (tax receipts, inflation erosion and real erosion of liabilities) whereas γ_2 attaches expenditure to nominal interest outlays, which vary counter-cyclically.

This is corroborated by *table 5.2* which gives ϑ_{max} for each of the regimes considered. This table brings out that the Christ regime performs best, closely followed by the Tobin-Buiter regime. The Barro regime, aiming at a constant debt ratio, is apparently the most procyclical; it yields the lowest ϑ_{max}.

Finally consider the impact of public debt on the system's stability. It is obvious that this impact is closely connected with the policy regime, especially with the reaction coefficient with respect to interest outlays (γ_2). If γ_2 is small, government debt will have a destabilizing impact as it increases the interest income of the private sector, thereby making consumption more pro-cyclical. However, when γ_2 is large, a larger public debt will stabilize the system. This is illustrated in *figure 5.7* which

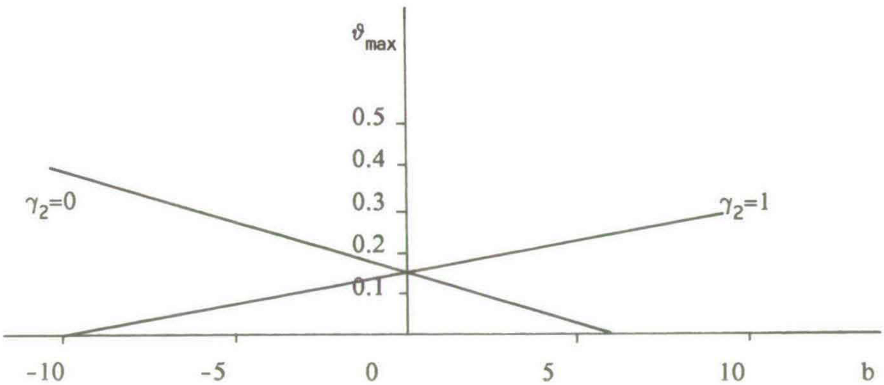
Table 5.2 ϑ_{max} for each regime

Blinder-Solow	0.151
Christ	0.172
Tobin-Buiter	0.168
Domar	0.149
Barro	0.127

Explanation: see table 5.1 above

gives ϑ_{max} in relation to government debt for γ_2=0 (as in the Blinder-Solow regime) and γ_2=1 (as in most other regimes). If γ_2=0, the maximum ϑ decreases as b goes up. At b=6.1 it even becomes zero, which implies that for any b>6.1 no stable steady state can exist whatever the adjustment speed of investment ϑ. If γ_2=1 the maximum for ϑ proves to rise with b. In this case b should not be too low (b>-10) for a stable solution to be feasible.

Figure 5.7 ϑ_{max} in relation to public debt



5.8 CONCLUSION

In this chapter we have examined the medium-period dynamics which is governed by disequilibrium between aggregate demand and supply and a 'Harrodian' interaction between demand growth and investment. It was shown that the introduction of monetary feedback may in principle stabilize the system, but only within a certain region, or '*corridor*', around the equilibrium point. Numerical simulations bring out that if the equilibrium is stable there may exist an *unstable limit cycle*; starting from a point within this cycle the system recedes to its equilibrium position, but whenever it starts from outside the cycle it will sooner or later come into a cumulative spiral of declining demand and investment.

Whether a stable solution exists depends primarily on the strength of the monetary feedback, the degree of price flexibility and the sensitivity of investment to changes in demand. Also fiscal policy proved to have a significant impact on the system's stability. In general, the system appears to be more stable as fiscal policy reacts more strongly to changes in debt service (γ_2 high), and less strong to changes in tax receipts, unemployment benefits and the size of public debt. Therefore the Barro and the Blinder-Solow regimes prove to be less stable then other regimes. The Christ regime emerges as the most stabilizing regime.

Besides, a small fiscal response to changes in debt service might also give rise to a *reverse interest effect*; in this case a higher interest rate leads to *higher* aggregate demand, as the result of a distribution effect in favour of workers. It is obvious that this effect may seriously undermine the stabilizing feedback of the monetary sector.

The impact of public debt on the system's dynamics depends on the fiscal policy regime. If the fiscal response to changes in debt service is small the system becomes less stable as debt is larger. Beyond some critical value the system proves to be unstable however low the accelerator factor of investment, and however high the flexibility of prices. Thus, although public debt is a slow variable in comparison to the demand variables that dominate medium-period dynamics, it may have an important impact in the long period. Gradual growth of public debt may sooner or later affect the stability of the system, and turn it from a (locally) stable system into an unstable system; that is, it may give rise to a 'catastrophe.'

In the next chapter we shall examine the dynamics of growth and asset accumulation from a long-term point of view. In contrast with the present chapter, the analysis will then concentrate on the dynamics of income distribution and the accumulation of public and corporate debt.

APPENDIX 5.A MEASUREMENT OF THE GOVERNMENT BUDGET DEFICIT

In our discussion of the alternative budgetary regimes in section 5.3 we followed the authors in their interpretation of the government budget constraint. There exist, however, several important conceptual difficulties with respect to the measurement of the budget deficit, especially with respect to the measurement of the *real* burden of debt. In its conventional definition the deficit on cash basis is given by

$$\text{deficit} = g - (T_1+T_2) + \tau_3 w(l_s - l) + (r+p)b \qquad (A.1)$$

Note that this deficit is not equal to the excess of expenditure g over income y_g, neither to the change in government liabilities (Dm+Db):

$$g - y_g \quad = \text{deficit} - p(m+b)$$

$$(Db+Dm) \quad = \text{deficit} - (i+p)(b+m)$$

The difference between the deficit and $(g-y_g)$ is the inflationary erosion of government liabilities, or the 'monetary correction' (Tanzi, Blejer, Teijeiro 1987).[21] The difference with the change in the ratio of government liabilities (Dm+Db) given by the government budget constraint (GBC) consists of the inflationary erosion and the 'real' erosion of liabilities as a result of growth of real income.

These results indicate that for a proper account of the burden of debt one should take nominal interest outlays net of devaluation of outstanding liabilities through inflation. In much IS/LM modelling of the GBC this problem is avoided as the price level is assumed to be fixed anyway (cf. Tobin, Buiter 1977 and Christ 1978, 1979). In a more complex analysis one cannot, however, neglect this problem.

Nevertheless, from a practical point of view, when modelling actual government behaviour it may be doubted whether the real or the nominal burden of debt offers a more relevant measure of the burden of debt. Politicians often seem to be more impressed by the nominal interest payments than by the less apparent benefits of devaluation of current liabilities (public debt and base money[22]) through inflation. This is further stimulated by the conventional 'book-keeping' methods used for accounting and budgeting. Especially in times of fiscal restraint the use of cash accounting methods, sometimes even in the form of explicit cash limits (cf. Rau 1985), favours a nominal bias in the budgetary regime.

Besides changes of the value of money, changes in the market value of public debt may also affect the real burden of debt. When the general level of interest rates falls or adverse shifts occur in portfolio-preferences towards public debt, the market value of outstanding public debt goes up, thereby raising the true burden for the government. This leads to very intricate difficulties with respect to the discount rates and time horizons of the public and the government.[23] In the present analysis we neglect these difficulties by assuming that all government debt has a fixed nominal value and a variable interest rate. Since it is further assumed that public debt and private sector debt are perfect substitutes, we may neglect portfolio shifts as well; there is only one homogenous sort of debt and one interest rate.

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[21] This inflationary erosion of government debt does not occur when the debt is indexed, or when the debt is nominated in foreign currency. Sometimes this erosion is considered as a form of amortization (cf. Tanzi, Blejer, Teijeiro 1987). Of course, these considerations do not apply to the erosion of money stock.

[22] Deficit finance is assumed to be the only source of base ('high powered') money.

[23] See Buiter (1983) and Tanzi, Blejer and Teijeiro (1987) for comprehensive accounts of these problems.

CHAPTER 6

LONG-PERIOD DYNAMICS OF GROWTH AND DEBT

6.1 INTRODUCTION

Despite these critical results with respect to medium-period stability, income-expenditure equilibrium with full-capacity utilization is taken as the starting point of the long-period analysis. This is necessary to concentrate on the dynamics of growth and asset accumulation, in particular the role of the government budget constraint in the long-term dynamics of the economy. As in the foregoing chapter we shall assess the stability of the system for different regimes of fiscal policy.

Since the seminal papers of Christ (1963) and Blinder and Solow (1973) many studies have been published on the dynamics of growing public debt and interest payments (cf. Tobin and Buiter 1976, Infante and Stein 1976, Turnovsky 1977, Christ 1978, 1979, Calvo 1985, Rau 1985, to mention but a few). Although these studies focus primarily on the effectiveness of fiscal and monetary policies, they have also yielded important results with respect to the stability of public debt and interest payments. One of the main conclusions common to these studies is the basically unstable nature of public debt when the deficit is financed by issuing interest-bearing debt.

The essential destabilizing element in this process is the interest payment on outstanding debt which has to be financed too, thereby further increasing the debt and so on. For the process to be stable it is required that other factors sufficiently neutralize this destabilizing tendency. In most models this is accomplished only if the income and wealth effects of the growing public debt on private expenditure are so strong that the resulting increase in tax revenue eventually exceeds the rise in interest payments. Since the income-effect of interest paid on public debt will never be sufficient to produce such an increase in tax receipts, it is evident that stability in these models hinges on the strength of the wealth effect of the growing public debt.

In the present chapter we shall challenge this negative view on the stability of public debt for a growing economy. In our view the studies mentioned above are not really suitable for answering the question of stability because they are restricted to a basically stationary IS/LM framework, in which autonomous expenditure, labour supply and often even the capital stock[1] are taken as fixed. Although there may often

[1] Cf. Blinder and Solow (1973) in their first model, Turnovsky (1976, 1977), Nguyen and Turnovsky (1979, 1983), Christ (1978, 1979), Calvo (1985), Rau (1985).

be good reasons to avoid the "complex issues of disequilibrium growth theory" (Rau 1985, p.214), we believe that for assessment of the stability of public debt it is indispensable to take account of the inter-relationship between the accumulation of public debt and the real growth of the economy. Confining the analysis to the (arbitrary) zero growth situation is in our view too restrictive; it obscures the basic factor determining stability, namely the growth rate of real income relative to the real interest rate.[2]

Some authors have in effect attempted to generalize the analysis for steady growth states, but so far little attention has generally been paid to the dynamics of public debt accumulation; either the analysis is restricted to the case of monetary financing (Infante and Stein 1980), or it focusses on the comparative statics of steady growth equilibrium (Tobin and Buiter 1980, Tobin 1980, 1982). In the recent literature there appears to be a tendency to evade the stability problem by assuming 'feedback' rules for government behaviour which make fiscal policy subordinate to a stable evolution of public debt from the outset (cf. Buiter 1986, Van de Klundert, Van der Ploeg 1987).

A more or less explicit account of the significance of growth for the dynamics of debt accumulation is given by Turnovsky (1978). For the case of money financing Turnovsky (p. 12) in fact concludes that the condition for (local) stability "is more likely to be met for a growing, rather than a stationary, economy." However, for the case of bond financing in which we are interested, his analysis is rather unsatisfactory, as for the stable system an increase in government expenditure should always lead to a *reduction* in public debt. According to Turnovsky (1978, p. 18) this "seemingly perverse" result is "largely a consequence of the fact that pure bond financing gives rise to continuously accumulating interest payments, thereby creating a highly destabilizing influence." In the present analysis it will be argued that in a growing economy a bond-financed budget deficit is not necessarily destabilizing at all. Moreover it will be shown that an increase in government expenditure may lead to a new stable equilibrium without any 'perverse' effects.

The analysis starts from the basic model given in the foregoing chapter. Section 6.2 completes this model for the long period and introduces a macroeconomic investment function based on the microeconomic analysis of the growth of the corporate firm in chapters 3 and 4. Section 6.3 analyses the long-term dynamics of public debt and income distribution for a reduced (two-dimensional) model concentrating on the accumulation of public debt and income distribution, while section 6.4 discusses the fully dynamic model including the interaction between investment and corporate debt.

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[2] Of course, the incorporation of growth in the model has its price too; in order still to be able to establish the determinants of stability we must adopt rather stringent restrictions on the modelling of demand dynamics and expectations. This has in fact been the basic motivation for the division of the analysis into the medium period and the long period.

6.2 THE LONG-PERIOD MODEL

As the medium-term analysis of the foregoing chapter has raised serious doubts with regard to the 'automatic' equilibrium restoring mechanisms, we shall assume that medium-term equilibrium is achieved by discretionary monetary policy which ensures that the money supply is always sufficient to generate full capacity utilization.[3]

The basic model is given by equations 5.1 to 5.17 in the foregoing chapter (section 5.2). For the long-term analysis it is completed as follows. First, notice that medium-term equilibrium, as we have seen, implies that capacity is fully utilized and that expectations on prices and demand are always fulfilled

$$y = y_c \tag{6.1}$$

$$p_e = p = p_o \tag{6.2}$$

As regards investment we shall now concentrate on the long-term determinants discussed in the chapters 3 and 4. It emerged from this microeconomic analysis that in the presence of imperfect markets for risk sharing, investment can be conceived as a function of the supply of internal savings and the desired debt ratio. The amount of internal savings is determined by the profit rate, the interest rate and the net pay-out to shareholders. The basic determinants of the desired debt ratio were found to be:

1. the probability distribution of profits and the interest rate;
2. the time preference and risk aversion of shareholders;
3. the preference towards growth and risk of managers;
4. the discretionary power of managers vis-à-vis shareholders.

In the present macroeconomic model shareholders and managers are taken together in the corporate class. This class owns all capital stock and reinvests its savings in the corporate sector in the form of equity. Workers are supposed to own no shares; they invest their savings in public or corporate debt (with a variable interest rate). This implies that in the present model internal savings of the corporate sector are equal to total sector income less consumption (y_2-C_2). Hence investment can be modelled as

$$i^* = \frac{y_2-C_2}{1-a} + \Omega\,\frac{a^*-a}{1-a} \qquad \Omega>0;\ a,a^*<1 \tag{6.3}$$

where

[3] As an alternative, fiscal policy can also be assumed to take care of demand equilibrium. In that case government expenditure or taxes should always be such that the budget deficit precisely compensates the excess of savings over investment by the private sector at the prevailing rate of interest. As this case is very similar to the balanced-current-account regime to be considered in the next chapter, it will be left out of consideration here.

$$a^* = a^*(\pi, r, \varsigma) \tag{6.4}$$

The first term in the investment function (6.3) represents the rate of investment compatible with a constant debt ratio[4]. As in chapter 3 this factor implies that the interest rate may have a *positive* influence on internal savings, and thus on investment, when the firm has a net creditor position. However, besides this income effect the interest rate also has a substitution effect on investment as it will probably change the desired debt ratio a^* in the second term. This second term represents the impact of the desired change in the debt ratio, modelled as a linear relation of the difference between the desired debt ratio a^* and its actual value a.[5]

In accordance with our non-'Modigliani-Miller' analysis of chapters 3 and 4 it is assumed that there exists a unique optimum for the debt ratio a^*, which is dependent on the profit rate and the interest rate (eq. 6.4). The state of risk and the preferences of shareholders and managers, as well as the discretionary power of managers are taken together in the exogenous factor ς.[6] Following Robinson (1962) this factor, which determines the locus of the investment function, can be interpreted as the 'animal spirits' of entrepreneurs. The maximum a^* is of course 1. In the numerical analyses following we shall use as a function for a^*:

$$a^* = 1 - \{\varsigma(1+\epsilon_1\pi-\epsilon_2 r)\}^{-1} \qquad \varsigma,\epsilon_1,\epsilon_2>0;\ 1+\epsilon_1\pi-\epsilon_2 r>0 \tag{6.4a}$$

This equation states that a^* is positively related to the profit rate π and 'animal spirits' ς, and negatively to the interest rate r. Entrepreneurs are thus supposed to accept a larger risk as the profit rate is higher and the interest rate lower. Equation (6.4a) has the plausible characteristics that $\lim_{\pi\to\infty} a^*=1$ and $\lim_{\pi\downarrow\pi_0} a^*=-\infty$, where $\pi_0=(\epsilon_2 r-1)/\epsilon_1$ represents the minimum for π. If the profit rate approximates this critical rate, entrepreneurs decide to invest all their wealth in financial assets (government bonds) and close down productive capacity.

Finally, we shall again adopt a linear modelling of the consumption function (see equation 5.20 in the foregoing chapter) and an explicit function for the interest rate[7],

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[4] This follows from the budget constraint (5.16).

[5] Notice the similarity between this function and Kalecki's function based on his principle of increasing risk (see section 3.2).

[6] Corporate taxes are assumed not to affect the optimum debt ratio. This requires all components of income of firms/capitalists to be taxed at the same rate. See also section 3.3.

[7] This equation for the interest rate follows from the demand for money function: $m_d = \mu_2 y/[\log\{(r+p-r_0)/\mu_1\}-\mu_3 b]$. This function implies that money demand increases if the interest rate falls and public debt rises.

$$r = r_o + \mu_1 e^{(\mu_2 y/m + \mu_3 b)} - p \qquad r_o,\mu_1,\mu_2,\mu_3>0 \tag{6.5}$$

This function has the attractive properties that $r \to \infty$ if $m \downarrow 0$ or $b \to \infty$, and that r falls into a liquidity trap (r r_o-p) if m becomes very large or b very low ($\ll 0$).

At a given production y and inflation p the interest rate can be manipulated freely by monetary policy. Although the total amount of government liabilities (m+b) is given at any moment, monetary authorities can instantaneously vary the mix of (base-) money and debt. The sum of money and debt (m+b) only changes as a result of the government budget deficit.[8] For simplicity, we do not distinguish an independent central bank, but include the central bank in the government sector. Monetary policy, aiming at equality between demand and capacity, takes place through open market policy, that is through changing the mix of m and b.[9] This is a consistent modelling of the relation between the government budget constraint and monetary policy within the context of our analysis.

Now the static part of the model is completed. For momentary equilibrium the stocks of corporate debt (a) and government liabilities (m+b) as well as the unemployment rate (u) and wages (w) are given. Then production is determined by the existing stock of capital and the optimum technique of production. Monetary policy manipulates the mix of total government liabilities and the interest rate in such a manner that aggregate demand always equals capacity.

Dynamics

The dynamics of the system is determined by the budget constraints of the government and the corporate sector (hereafter abbreviated as GBC and CBC respectively) and by the evolution of the wage rate and the rate of unemployment. Let z represent total government liabilities (=m+b), so that the GBC reduces to

$$Dz = -y_g + g - i.z \tag{6.6}$$

The CBC was already given in the foregoing chapter (eq. 5.16),

$$Da = -y_2 + i + C_2 - i.a$$

---

[8] As monetary policy must maintain equilibrium between demand and supply, the mix of debt and money is an endogenous variable in the present model unlike in most (medium-term) analyses of the GBC which generally assume a specific rule for the money-debt mix.

[9] In reality monetary policy may also be pursued by regulations or by manipulating the tariffs on central bank facilities. As in most theoretical analyses we take open market policy as the typical instrument of monetary policy.

As monetary policy ensures demand equilibrium and a constant inflation (p_o) it is natural to assume that the evolution of real wages depends on labour market disequilibrium:

$$Dw = f(Du, u-u_o) \qquad f(0,0)=0;\ f_1,f_2 \leq 0 \qquad (6.7)$$

where u = unemployment rate; u_o is the unemployment rate compatible with a constant income distribution.[10] The change in unemployment (Du) is found by differentiation of u with respect to time:

$$Du = (1-u)(n - \hat{l} - i) \qquad 0<u<1 \quad (6.8)$$

where n stands for the growth of labour supply (in efficiency units) and $\hat{l}$ for the change in labour intensity. Before further discussing the dynamics of this full model, we shall as a first approximation look at a reduced, two-dimensional, version of this model.

6.3 LOCAL AND GLOBAL STABILITY

In order to reduce the dimensionality of the model we shall concentrate on the dynamics of two state variables: income distribution (represented by the profit rate π) and government liabilities (z). For the moment corporate debt and labour intensity are taken to be constant. Also the dynamics of the unemployment rate is neglected. Therefore we assume that no unemployment benefits are paid, so that the budget deficit is independent of u, and that the dynamics of income distribution is given by

$$D\pi = \vartheta_\pi(n - i) \qquad \vartheta_\pi>0 \qquad (6.7a)$$

According to this function the profit rate falls when the labour market tightens, and rises when unemployment increases. Together with the GBC (eq. 6.6) this equation determines the dynamics of this reduced model.

Unfortunately the model is still too complex to handle on an analytical level. Therefore, we shall proceed with some numerical exercises for a plausible parameter set. The analysis which folows is based on this reference set of parameters:

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[10] This rate is neither the natural rate of unemployment nor the Keynesian equivalent, the NAIRU (Non Accelerating Inflation Rate of Unemployment, cf. Cornwall 1983), but the unemployment rate which is compatible with a constant distribution of income, i.e. constant shares of wages and profits, or in 'new speak' economics: the 'CIDRU' (Constant Income Distribution Rate of Unemployment).

r_o=0.02; p_o=0.02; g/y=0.22; $\tau_o=\tau_1=\tau_2$=0.2; τ_3=0; γ_1=0; γ_2=0; γ_3=0; γ_4=0; a=0.5; c_1=0.9; c_2=0.6; c_{1r}=-1; c_{2r}=0; c_{1z}=0.05; c_{2z}=0; c_{1o}=-0.016; c_{2o}=0; ς=1; $\epsilon_1=\epsilon_2$=12; μ_1=0.0001; $\mu_2=\mu_3$=1; l=0.3; β=0.8.[11]

The basic properties of the long-period model with these parameters can be established from the following partial effects:

Table 6.1 Some sensitivities

wealth effect on aggregate savings:	dS/dm = -0.05
profit effect on aggregate savings:	dS/dπ = 0.24
interest effect on aggregate savings:	dS/dr = 0.52
interest effect on investment:	di/dr = -1.1
profit effect on investment:	di/dπ = 1.3
money elasticity of interest rate:	dr/(dm/m) = -3.8
debt elasticity of interest rate:	dr/(db/b) = 0.15

Explanation: These effects have been evaluated at the steady state solution to be discussed below: π=0.11; b/y=0.58; m/y=0.17: n=0.05; r=0.04

where aggregate savings (S) are defined including government savings (thus $S=y-C_1-C_2-g$). These results are in line with general empirical evidence.[12] As we have already observed in the foregoing chapter the effect of the interest rate on savings is very sensitive to the budgetary regime, especially to the reaction coefficient with respect to interest outlays (γ_2). Moreover, it varies with the size of public debt (and thus interest-income of the private sector). This is corroborated by *table 6.2* which gives the numerical results for a Blinder-Solow regime (γ_1,γ_2=0) and a Domar or Barro regime (γ_1,γ_2=1) for varying ratios of public debt.

In the case of the Blinder-Solow regime (γ_1,γ_2=0) the interest effect on savings proves to fall as debt grows. For a large debt (in the present example $b\geq 1$) the interest effect even becomes negative; a higher interest rate then leads to an *increase* in consumption rather than a decrease. This is due to the distribution effect which becomes stronger as the flow of interest payments from the government to the private

[11] As for the corporate class savings are a means of increasing the size of the capital stock rather than deferring consumption, it is difficult to assess the wealth and interest elasticity of consumption. This should be based on a fully-fledged microeconomic analysis of simultaneous consumption, investment and portfolio behaviour of the corporate class. This is, however, beyond the scope of the present analysis. Therefore we have chosen to neglect the impact of the interest rate and the profit rate on the consumption of the corporate class, and to include them in the functions for investment and desired debt.

[12] Following Modigliani (e.g. Modigliani 1971), the wealth effect on consumption is remarkably often found to be -0.05 (for the Netherlands see e.g. the quarterly model of De Nederlandsche Bank 1985). The result for the money elasticity of the interest rate implies a semi-interest-elasticity of money demand of -0.25 which is in accordance with empirical evidence (cf. Judd and Scadding 1982).

Table 6.2 Interest effect on aggregate savings (in points)

public debt (b)	-1	0	1	2	4
$\gamma_1,\gamma_2= 0$	1.4	0.7	-0.04	-0.76	-2.2
$\gamma_1,\gamma_2= 1$	0.6	0.7	0.76	0.84	1.0

sector are larger. For the Domar or Barro regime ($\gamma_2,\gamma_1=1$) the interest effect proves to be practically constant, only increasing slightly as b goes up. In this case the distribution effect is much weaker: the positive effect of higher interest payments on private consumption is now offset by the simultaneous reduction of government expenditure.

The non-loglinear modelling of portfolio equilibrium (eq. 6.5) implies a variable elasticity of the interest rate with respect to money and debt. The numerical results in *table 6.3* indicate that the impact of public debt on the interest rate becomes stronger as the size of debt increases. This is in accordance with the empirical notion that the influence on the interest rate becomes significant only for a large debt. As a corollary the impact of money on the interest rate also becomes stronger as debt is larger in relation to the money supply.

Table 6.3 Semi-elasticities of the interest rate

public debt (b)	-1	0	1	2	4
dr/(dm/m)	-2.2	-3.5	-4.6	-5.2	-5.5
dr/(db/b)[13]	-0.4	0.001	0.8	1.9	4.0

Dynamics

The steady-state solution of this system is found at $d\pi=0$, and thus i=n, and Dz=0. This latter condition requires that the government budget satisfies

$$(g-T) + z.[\{(1-\gamma_2)-(1-\gamma_1)\tau_1\}r - (\gamma_2-\gamma_3)p - (1-\gamma_4)i - \\ - \{(1-\gamma_2)-(1-\gamma_1)\tau_1\}(p+r)m/z] = 0$$

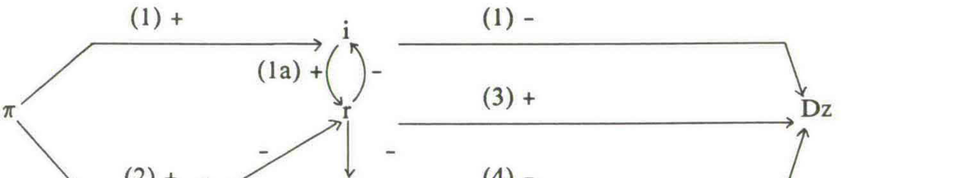
[13] For negative debt the elasticity is, of course, negative.

The first term in this equation represents the primary deficit (g-T) and the second term the impact of the interest rate and the growth rate on Dz. For a 'normal' solution with a positive primary deficit and a positive steady state value of z, it can be seen that the term between square brackets should be negative. For the Blinder-Solow regime (all γ's zero) this would require $-\{i-(1-\tau_1)r\}+(1-\tau_1)(r+p)m/z>0$, and thus if m is sufficiently small in relation to z: $i-(1-\tau_1)r>0$. This result that the growth rate should exceed the real interest rate net of taxes is, however, not valid in general. It can easily be seen that, for example, the Christ regime ($\gamma_2=1$) yields (for a small m): $i+\tau_1 r>0$ which is satisfied for any positive i and r. This makes it clear that no simple conclusions can be drawn concerning the relation between the growth rate and the interest rate in steady state equilibrium. We shall therefore not dwell on the steady state too long, and now turn to the primary interest of this analysis: the dynamics that follows from the differential equations for z and π.

First consider the determination of Dz by π and z. After substitution for y_g in equation (6.6) the GBC becomes

$$Dz = g + (1-\tau_1)rz - \tau_1 y - (\tau_2-\tau_1)(\pi-ra) - (1-\tau_1)(r+p)m - i.z \qquad (6.6a)$$

If we neglect the consequences of possible differential taxing of profits and wages ($\tau_2=\tau_1$) it can assessed that the overall effect of the profit rate π on Dz depends on several opposite effects given in the scheme below:



First, a higher profit rate leads to a higher growth rate (i) and thereby to a smaller change in the liability ratio z (linkage 1 in the scheme); this negative 'real erosion' effect is larger in absolute terms as z is larger. This effect is, however, weakened by the rise in the interest rate associated with the increase in investment (linkage 1a). Secondly, a higher π increases savings (distribution effect) and thereby lowers the interest rate; this reduces the burden of interest payments and pushes the growth rate up, both factors leading to a lower growth of z (linkage 2). Note that the effect of the lower interest rate is reinforced by the corresponding rise in money stock, and thus fall in debt, which also reduces debt service (linkage 4).

As the real erosion effect of the higher growth rate (linkage 1) becomes stronger as z increases while the other effects are independent of z, it can be concluded that the overall effect of π on Dz will always be negative for a sufficiently large z. Hence there will exist a critical size of liabilities z_o for which

$$\partial Dz/\partial\pi > 0 \text{ for } z < z_o \text{ and } \partial Dz/\partial\pi < 0 \text{ for } z > z_o$$

Next consider the impact of the amount of liabilities z on its rate of change Dz. The overall impact consists of the following effects[14] (all effects are discussed for a deficit position and a given stock of money m):

1. *scale effect*: the impact of a given budget deficit on the growth rate of z is smaller as the initial size of z is larger (the term -i.z in equation 6.6a);
2. *budgetary effect*: a larger z increases debt service and thereby the budget deficit. This effect is mitigated by higher tax receipts on the interest income of workers and by the adjustment of government expenditure in reaction to the higher interest payments (if $\gamma_2>0$). The overall effect of a change in z on the budget is, after substitution for g, equal to $[\{1-\gamma_2-(1-\gamma_1)\tau_1\}r-(\gamma_2-\gamma_3)p+\gamma_4 i]z$.
3. *income and wealth effects*: a rise in z leads to an increase of private consumption through the increase in wealth and (interest) income. Both effects push up the interest rate and thus lead to a higher deficit and a larger Dz.
4. *portfolio effect*: at a given stock of money the larger z leads to a higher interest rate and thus to a larger Dz.

The two first effects represent the direct effects of z on the government's budget; the other effects affect the budget more indirectly through their effect on the interest rate. Since these indirect effects always lead to a larger Dz, it can be concluded that the direct effects should at least be negative if the overall effect is to be negative. Therefore

$$-(1-\gamma_4)i + \{1-\gamma_2-(1-\gamma_1)\tau_1\}r - (\gamma_2-\gamma_3)p < 0 \tag{6.9}$$

is a necessary (but not sufficient) condition for $\partial Dz/\partial z<0$. For the conventional Blinder-Solow regime (all γ's zero) this condition again yields the requirement that the growth rate should exceed the real interest rate after taxes ($i-(1-\tau_1)r>0$). For other regimes this condition is considerably relaxed as these entail a negative feedback of debt service on government expenditure ($\gamma_2>0$).

As the interest rate rises with the volume of government liabilities it can be

[14] Rau (1985) distinguishes four effects of public debt: transfer, wealth, money-market and Friedman effects. His transfer effect corresponds to our income effect, his wealth effect to our wealth effect, and his money market effect is what we prefer to call the portfolio effect. His Friedman effect (called after Benjamin Friedman) refers to the positive effect of increasing debt on share prices and thus on investment; this effect, which arises from imperfect substitution between debt and equity, is neglected in the present analysis as we have consolidated the corporate sector with its owners and shareholders.

established that the overall impact of z on Dz will be positive (and thus de-stabilizing) for a large z, thus

$\partial Dz/\partial z > 0$ for a large z

When z is small it is probably negative.

All effects have been discussed with respect to the deficit situation (z,b>0). It is obvious that when the government is a net creditor the conclusions will be quite different, particularly because the scale and income effects change direction. When b<0 a higher interest rate *improves* the position of the government so that Dz *decreases*, etc. As this case is of little relevance from a practical point of view we shall leave this case as it is, and concentrate on the conventional debtor case.

The phase diagram

Now consider the dynamics of this model. *Figure 6.1* presents a phase diagram of this model in the $\{\pi,z\}$ plane, showing the conditions for stable government liabilities (Dz=0) and income distribution ($D\pi=0$). The steady state growth is found at:

$\pi=0.11$; b/y=0.58; m/y=0.17; n=0.05; r=0.04

(In order to ease the interpretation of these results, public debt and money stock are expressed in relation to national income y).

The shape of the Dz curve in the phase diagram follows directly from the discussion above: for the reference parameter set the critical size of liabilities z_o where $\partial Dz/\partial\pi=0$ is found to be negative.[15] This critical z_o is represented by an asymptote in the figure. As we wish to concentrate on the debtor case we shall not bother about the region to the left of the asymptote ($z<z_o$). In the relevant region the Dz=0 curve can be seen to fall for low z when the scale effect is dominant (hence $\partial Dz/\partial z<0$), and to rise beyond some z when the interest rate becomes high ($\partial Dz/\partial z>0$).

The shape of the $D\pi=0$ curve can be explained as follows: As the interest rate rises with the size of public debt, the profit rate must go up too in order to maintain growth equilibrium. Hence the positive slope of the $D\pi=0$ curve. Further, as the impact of public debt on the interest rate becomes stronger as debt increases, the curve becomes steeper for high z's.

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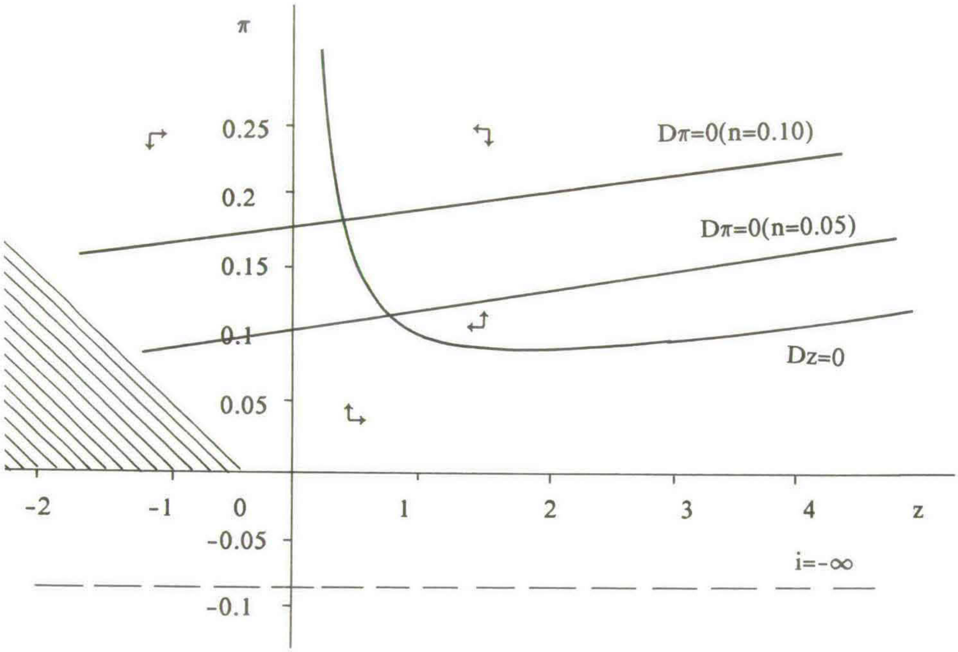
[15] This is due to the apparently strong distribution effect of π which implies that at z=0 higher profits raise savings more than investment, so that the interest falls. Therefore $\partial Dz/\partial\pi<0$ at z=0, and thus a critical $z_o<0$.

Figure 6.1 Phase diagram for a Domar regime ($\gamma_1,\gamma_2,\gamma_3=1;\gamma_4=0$)[16]



Explanation: This figure is based on the reference set of parameters given above.

Figure 6.1 gives the phase diagram for a Domar regime ($\gamma_1,\gamma_2=1$). In the given range this model yields only one solution, which can be seen to be stable within a fairly wide area. The figure also exhibits a lower boundary for the profit rate at π=-0.09. This arises from the bottom in the interest rate function (liquidity trap); if π approaches this boundary the interest rate can no longer fall sufficiently to avoid total disinvestment of production capacity ($i=-\infty$). In the shaded area at low z (<<0) no momentary equilibrium can be found because of inconsistency between saving and investment for any possible monetary policy. In this region private income has become so low and government income so high (as a result of the large interest payments by the private sector *to* the government) that savings exceed investment even at the

[16] Throughout the following analysis we assume that the government has no inflation illusion and treats inflationary effects on debt and money stock on the same footing as nominal interest payments, hence $\gamma_3=\gamma_2$.

lowest possible interest rate.[17]

In the figure two curves for $D\pi=0$ are shown, one for a natural growth rate n=0.05 and one for n=0.10. The latter curve lies, of course, above the first as a higher growth rate of course requires a higher profit rate. The $Dz=0$ curve is independent of n. The figure brings out that a higher natural growth rate yields a higher steady state profit rate and lower liability ratio. At both growth rates the system can be seen to be thoroughly stable.

Blinder-Solow regime

This general stability may change drastically for other budgetary regimes. *Figure 6.2* presents the results for the Blinder-Solow regime with fixed government expenditure ($\gamma_1,\gamma_2=0$) in *fig.a*, and for an intermediate regime ($\gamma_1=0;\gamma_2=0.5$) in *fig.b*. Comparison of these figures brings out that, although neither the steady state nor its local stability changes, the system is considerably less stable when the fiscal response to interest payments (γ_2) is low. Especially for initial positions with high debt and low profits there is a great risk of a cumulative process of growing debt and interest payments.

This figure further brings out that there may exist a second restpoint. This solution is, however, characterized by a saddle-point configuration, and is thus unstable. Therefore, if the system starts from the left of the dashed A-A curve it tends to the stable solution (S), but whenever it starts at a point at the right of this separatrix it falls into an unstable spiral of ever rising z and π.[18]

Note that the two solutions practically coincide for the Blinder-Solow regime ($\gamma_1,\gamma_2=0$) if the natural growth rate is 5%. In this case the system is thus stable only for upward disturbances in π and for downward disturbances in z. A significant fall in π or rise in z will lead the system away from the equilibrium for ever.

Finally, the figure demonstrates that the stability of the system is improved when the natural growth rate is higher. Although the basic asymmetrical instability remains, the region within which the system is stable is considerably larger if n=10% than if n=5%. If the growth rate falls below 5% the steady state solution even disappears altogether. These results show that the dynamics is very sensitive to the height of the natural rate of growth. Therefore there exists some critical minimum for the growth rate below which the system changes from a (locally) stable system into a totally unstable system.

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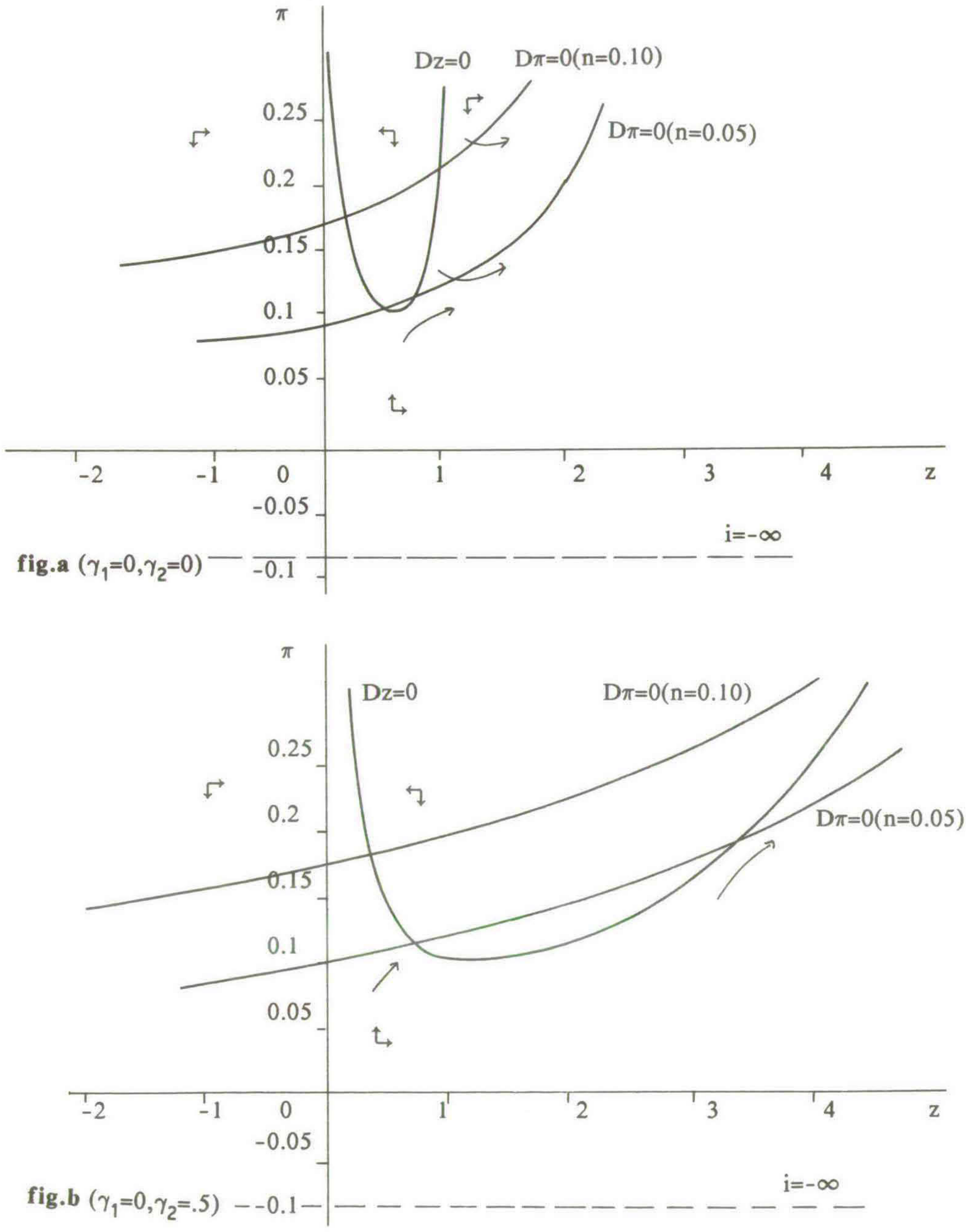
[17] Further increases in money supply do not resolve this inconsistency as total private sector wealth does not change; an open-market purchase reduces public debt by the same amount as it increases money. As a result, interest income of the private sector falls even further so that it may even lead to a rise in aggregate saving. In this situation only fiscal policy can resolve the inconsistency by increasing government expenditure or by raising private sector income through higher transfers or lower taxes.

[18] This process must sooner or later lead to a breakdown of the system as π cannot rise infinitely (contrary to what is implied by the linear function for $D\pi$ (eq. 6.7)).

Figure 6.2 Phase diagram for the Blinder-Solow regime (γ_1=0,γ_2=0) and an intermediate regime (γ_1=0,γ_2=0.5)

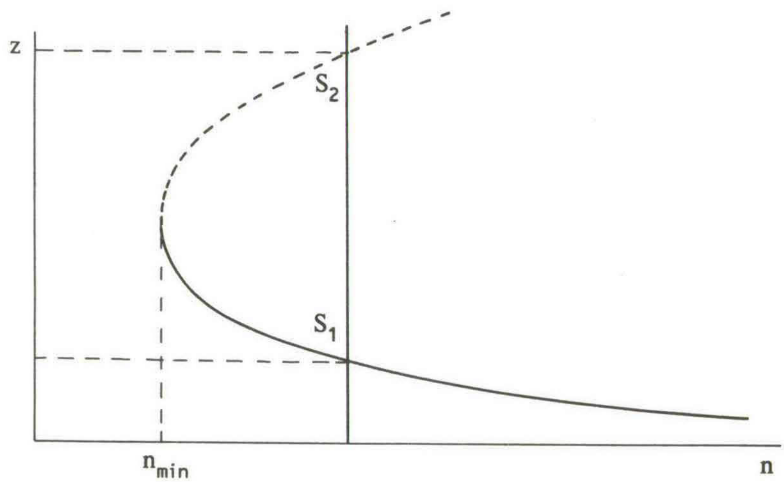


Explanation: see figure 6.1 above.

Catastrophe manifold

It may be noted that this minimum growth rate represents the bifurcation point of the catastrophe manifold of this system.[19] This is shown in *figure 6.3*. In fact, this figure depicts the equilibrium points for z in relation to n. As we have seen above, the equilibrium points on the upper branch are unstable and those on the lower branch stable. For $n<n_{min}$ no equilibrium exists.

Figure 6.3 Catastrophe manifold of the fold catastrophe



As before we can use this critical value of n to assess the impact of different variables and regimes on the (local) stability of the system. The results for a selection of the variables of the model are presented in *table 6.4*. A variable is stabilizing if it reduces the critical n and destabilizing if it raises the n required for a stable equilibrium.

As might be expected higher propensities to save and a lower autonomous part of the real interest rate (r_o-p_o) lead to a more stable system. As in the case of in chapter 2, the wealth effects on consumption turn out to reduce the system's stability, in contrast with most IS/LM based models where, as already mentioned, strong wealth effects are essential for stability. With regard to fiscal policy we find that stability can be improved by raising taxes on wages and the interest income of workers. A rise in taxes on profits, however, worsens the system's stability. This is obviously due to the

[19] This is a catastrophe manifold for a 'fold catastrophe' which - as we have seen in chapter 2 - produces a rather smooth catastrophe, unlike, for example, the more typical 'cusp catastrophe' which produces a sort of jump of the system.

Table 6.4 Partial effects on n_{min}

g_o	2.77	c_1	0.156
γ_1	0.026	c_2	0.015
γ_2	-0.073	c_{1r}	0.026
γ_3	0.011	c_{2r}	0.013
γ_4	0.048	c_{1z}	0.678
τ_0	-1.302	c_{2z}	0.203
τ_1	-0.089	c_{1o}	0.565
τ_2	0.283	c_{2o}	0.385
r_o	0.009		
p_o	-0.134		

Explanation: all effects are measured with reference to the intermediate regime given in figure 6.2. The effects of the γ coefficients have been corrected for their impact effect on government expenditure. One should be careful to compare the absolute effects because an equal change in each variable may entail quite different impacts in absolute terms.

negative impact on the growth rate. Also autonomous government expenditure proves to have a strong destabilizing impact. With regard to the budgetary regime it can be seen that the reaction coefficient with respect to interest payments (γ_2) has a stabilizing effect, while the other fiscal reaction coefficients tend to reduce the system's stability. Although the magnitude of these effects may change, the pattern of signs of the effects proves to be robust for other numerical simulations in the neighbourhood of the reference model.

All these partial effects have been measured with reference to a single intermediate regime with partial adjustment of government expenditure to changes in interest payments (γ_2=.5). Another way of looking at the determinants of stability is to compare the alternative regimes. *Table* 6.5 gives the minimum growth rates for each of the regimes considered:

Table 6.5 Minimum growth rate (%) for alternative policy regimes

Blinder-Solow	4.8
Christ	-1.2
Tobin-Buiter	0.9
Domar	0.3
Barro	stable for any n

Explanation: see table 6.4 above.

These results corroborate our inferences above: a strong budgetary reaction to changes in interest payments as in the Christ regime and, to a lesser degree, in the Tobin-Buiter regime improves the stability of the system. The Blinder-Solow regime is the least stable. The Barro regime is stable for any growth rate, which is hardly surprising as it presupposes that the government aims at a constant debt.

6.4 LONG CYCLES

In addition to the dynamics of income distribution and government liabilities the fully dynamic model takes account of the dynamics of unemployment and corporate debt as well. For generality the model also incorporates the possibility of slow adjustment of investment and government expenditure. Then choosing explicit functions the full dynamic system becomes:

$$Da = -y_2 + i + C_2 - i.a$$

$$Dz = -y_g + g - i.z$$

$$Du = (1-u)(n - \hat{l} - i) \qquad 0<u<1$$

$$D\pi = \{\vartheta_{\pi 1}Du + \vartheta_{\pi 2}(u-u_o)\}/u \qquad \vartheta_{\pi 1},\vartheta_{\pi 2} \geq 0;\ u_o>0 \qquad (6.10)$$

$$D^2a^* + \vartheta_{a1}Da^* = \vartheta_{a2}(a^{**}-a^*) \qquad \vartheta_{a2}>0 \qquad (6.11)$$

$$Dg = \vartheta_g(g^*-g) \qquad \vartheta_g>0 \qquad (6.12)$$

$$Di = \vartheta_i(i^*-i) \qquad \vartheta_i>0 \qquad (6.13)$$

where now a^{**}, g^* and i^* are given by the equations for desired a^*, g and i (eq. 6.4a, 5.11 and 6.3) above. The first two equations restate the budget constraints for the corporate sector and the government (eq. 5.16 and eq. 6.6). The third equation again gives the differential equation for the unemployment rate (eq. 6.8).

Equation (6.10) relates the change in income distribution to the evolution of the unemployment rate. For $\vartheta_{\pi 1}=0$ and $\vartheta_{\pi 2}>0$ this function is akin to the relation Goodwin (1972) uses in his famous model of the Marxian business cycle. The modelling of this relation is such that $D\pi$ tend to infinity when u becomes zero. This is in accordance with Goodwin's 'ideal function' (1972, p.444), although at variance with the linear approximation function he uses in his actual model. We prefer the non-linear 'ideal' function; moreover, it has the convenient implication that it excludes the possibility of absolute labour shortage ($u<0$).

The change in the desired debt ratio a^* is now modelled by a second order differential equation in accordance with our view that norms on proper debt ratios change only very slowly (eq. 6.11). If $\vartheta_{a1}=0$ this function implies a second-order adjustment, i.e. also the rate of change in a* can only change gradually over time. that The two last equations are partial adjustment functions for g and i. Because the slow reaction of wages to labour market imbalances is the prime cause of unemployment, the choice of technique is only of secondary importance to the dynamics of our model. For simplicity it is therefore taken to be constant in our numerical exercises below (hence $\hat{l}=0$).

At the given parameter values of the reduced model considered above, the fully dynamic model yields a basically similar steady state solution: m/y=0.17; b/y=0.77; π=11.0%; r=4.8%. The minor differences arise from the fact that the desired debt ratio is endogenous now with a steady state value of 0.43 instead of the arbitrarily chosen rate of 0.50 in the reduced model above. In order to gain some insight into this complex model we shall discuss some numerical simulations of this model. All simulations are made with reference to the Christ regime as this regime ensures the dynamics to be stable. It needs however little imagination to understand that other regimes, especially the Blinder-Solow regime, may produce trajectories that lead away from steady state equilibrium for ever (these unstable trajectories prove to be very similar to the trajectory shown in *figure 2.6* of chapter 2).

A real cycle

By way of reference, first consider a simulation of a 'real' Goodwin-like cycle (with $\vartheta_{\pi 2}>0$) where the interaction between the labour market and income distribution is the basic cyclical mechanism.[20] In order to isolate this cycle from the financial dynamics to be considered below we shall assume that desired debt is exogenous ($\vartheta_{a1}, \vartheta_{a2}=0$). *Figure 6.4* shows the evolution of public and corporate debt (*fig.a*) and profit, growth and interest rate (*fig.b*) after a combined shock at t=5. This shock represents on a very abstract level the rupture in economic growth of the 1970's which was caused by

1. a fall in profits, as a result of wages claims and oil price rises;
2. a decline in the natural rate of growth, through the slow-down of productivity growth (and population growth);
3. an adverse shift in business confidence ('animal spirits') leading to more prudential financial policies, as a reaction to the growing uncertainty and the 'over-optimism' of the 1960's.

This combined economic shock caused a structural slow-down of investment and economic growth. In our model this shock is represented by a discrete fall in π by 4 points, a permanent decline in n from 5% to 3% and a once and for all fall in ς by 20%. This latter shift in 'animal spirits' implies a discrete fall in the desired debt ratio a** from 0.43 to 0.28.

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[20] See Van der Ploeg (1983) for a sophisticated analysis of the Goodwin cycle in a continuous time model.

Figure 6.4 A real cycle (deviations from the initial equilibrium)

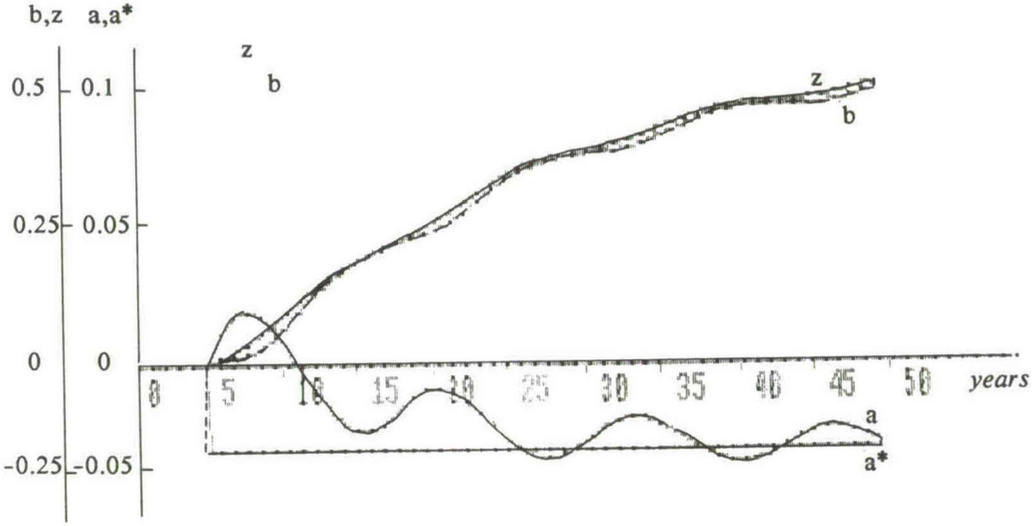

fig.a

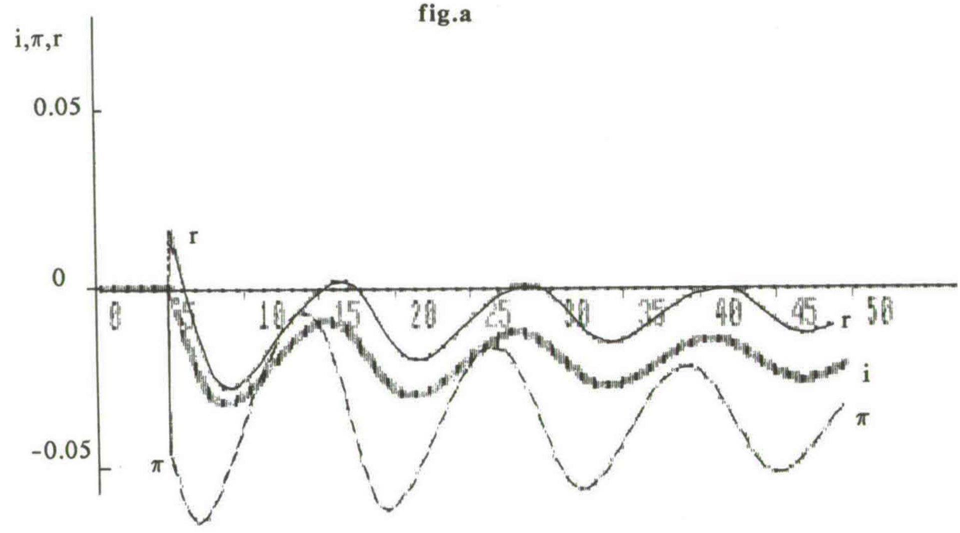

fig.b

Explanation: in addition to the parameters of the reduced model this figure is based on: $\Omega=0.1$; $\vartheta_i=0.5$; $\vartheta_g=0.3$; $\vartheta_{\pi 1}=0.05$; $\vartheta_{\pi 2}=0.01$; $\vartheta_{a1}=0$; $\vartheta_{a2}=0$. As to the policy regime it is assumed that $\gamma_2=1$ and all other γ's are zero (the Christ regime). The simulation is based on a numerical approximation of the differential system by the Runge-Kutta method (see e.g. Cohen 1973).

The figure shows the evolution of each variable as (absolute) deviation from its initial steady state value. It can be seen that the shock gives rise to a (dampened) cyclical movement, eventually leading to a larger government debt and smaller corporate debt, and a lower growth rate and profit rate. The difference between the new steady state and the initial equilibrium are summarized below

	initial equilibrium	new steady state
i(%)	5.0	3.0
π(%)	11.0	7.7
r(%)	4.8	3.3
a	0.43	0.28
m/y	0.17	0.19
b/y	0.77	1.32

After the shock at t=5 growth declines because of the fall in profits and the adverse shift in 'animal spirits.' As a result unemployment starts to grow and wages to fall; after some time profitability therefore recovers leading to a new expansion of investment and employment. Sooner or later this expansion will be checked by the tightening of the labour market as a result of which wages rise again and profits fall. This is the basic cyclical mechanism of Goodwin's Marxian cycle. Unlike the original Goodwin model, the present model incorporates a cyclical movement of the interest rate too, which obviously has a dampening influence on the cycle.

As regards corporate debt it can be seen that the fall in profits at t=5 initially leads to a rise of corporate debt, because investment does not adjust instantaneously to the lower internal savings. Thereafter, as investment falls and profits recover, corporate debt declines and tends cyclically to its new (lower) steady state level. At the same time public debt grows to a higher structural level, corresponding to the lower natural rate of growth.

Regarding the duration of these processes one may note that this cycle refers to the long 'Kuznetz' cycle, or even the Kondatrieff cycle, rather than the conventional (short) business cycle. The Kuznetz cycle, or 'long swing' is generally thought to have a length of 10-15 years whereas the conventional (American) cycle has a length of only some 5 years. The (non-existing)[21] Kondratieff wave even has an alleged duration of some 40-60 years.

[21] This emerges from most of the empirical research on the Kondratieff cycle (see e.g. Van Ewijk 1981, 1982a). Nevertheless, some investigations seem to yield more favourable results with respect to the existence of this long wave, see e.g. Metz (1984), Reijnders (1988).

A financial cycle

As an alternative to this 'real' explanation of the business cycle, our model also offers a 'financial' explanation, i.e. a cycle arising from corporate policies with regard to finance and investment. In order to concentrate on the financial dynamics now let $\vartheta_{\pi 2}=0$, thus eliminating the 'Goodwin' propagation mechanism.

First, note that the ultimate effects of the combined shock are practically the same for the financial cycle as for the real cycle given above. The steady state is now found at n=3%, π=8.1%, r=2.9%, a=0.22, m/y=0.19 and b/y=1.34. Only the result for the debt ratio (a) is significantly different; this is obvious as for the real cycle a* was assumed to be fixed.

Next consider the adjustment process. Just as in the real cycle above, corporate debt rises initially after the shock, as investment adjusts slowly to the lower internal savings. Only after some time, when investment is cut down sufficiently, does the rise in the debt ratio (a) turn into a decline. In the meantime the desired debt ratio (a*) has fallen considerably. While investment and actual corporate debt are still falling, desired debt starts to increase again as a result of the lower interest rate and the higher profit rate (due to the growing unemployment). Thereafter, when corporate financial policy is relaxed, investment and growth will recover too. After some time the growth rate rises again above the (lower) natural growth, so that the labour market starts tightening again. This leads to a fall in the profit rate, thereby pressing down the desired debt ratio again, inducing more restrictive corporate financial policies and thereby lower investment, etc.

This interaction gives rise to a clear cyclical pattern in the actual and the desired corporate debt ratios. The financial cycle may thus be interpreted as a succession of waves in the readiness to take risks and incur debts ('animal spirits'), or in Schumpeterian terms, by waves in business optimism and pessimism. Eventually the debt ratio will tend to a lower long-term level corresponding to the structural fall in growth and the shift in 'animal spirits.' As can be seen in the figure this process causes the growth rate to oscillate towards its new (lower) steady state value of 3%. The interest rate moves more or less parallel; the profit rate appears to lead by a couple of years.

Now consider the evolution of government liabilities. As a result of the slow-down in economic growth, government liabilities begin to grow, at first largely in the form of money (due to the expansionary monetary policy necessary to maintain sufficient demand), but later mainly in the form of debt. Eventually z tends to a new steady level of 1.54. This higher government debt is the natural consequence of the slow-down in growth.[22]

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[22] For other regimes, or other parameters sets, the system may, of course, be unstable. Then, government liabilities keep growing for ever.

Figure 6.5 A financial cycle (deviations from initial equilibrium)

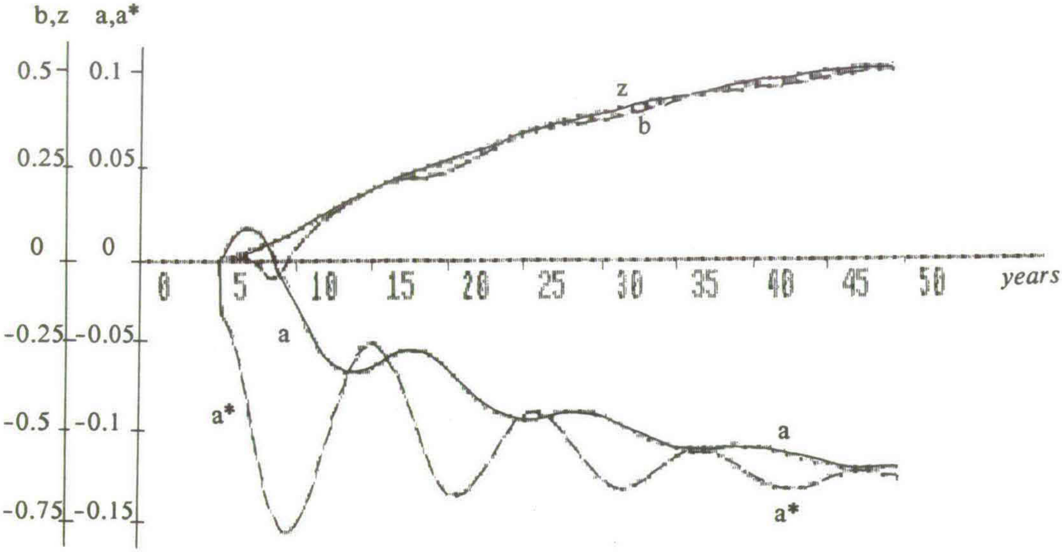
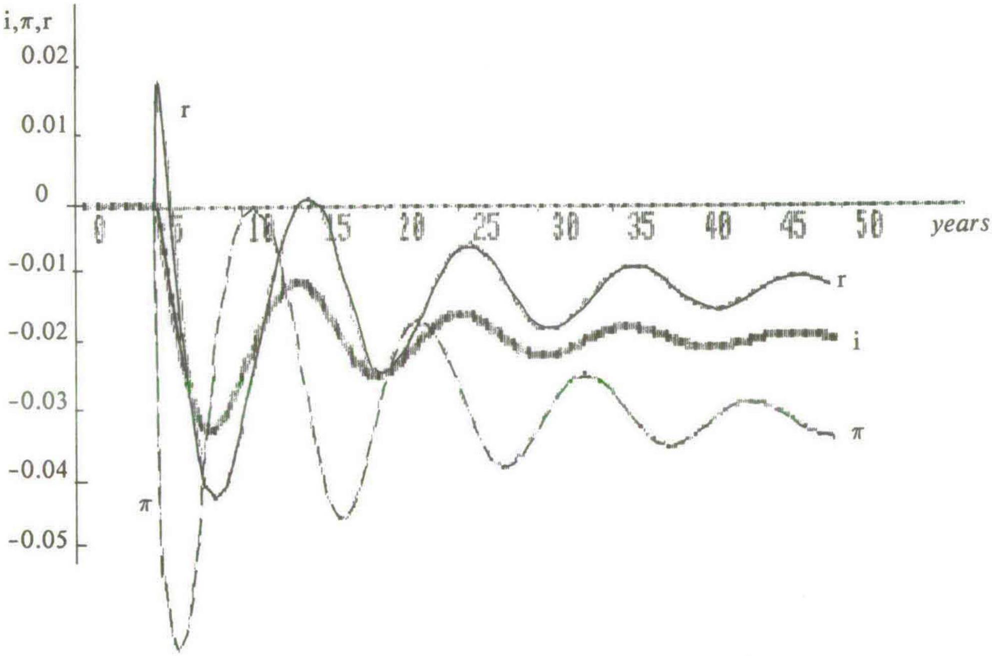


fig. a



Explanation: $\vartheta_{\pi 1}$=0; $\vartheta_{\pi 2}$=0.02; ϑ_{a1}=0.5; ϑ_{a2}=0.1 and all other parameters as in figure 6.4 above.

The appearance of this 'financial' cycle is not really different from the real cycle above. There is, however, a qualitative difference as regards the causation of the cycle: in the real cycle the principal propagation mechanism arises from labour market disequilibrium and income distribution, while the financial cycle is governed by the discrepancy between the actual and the desired corporate debt. Of course, no model is exclusively valid: both models clarify a certain aspect of the true dynamic system.

Steady state effects

Finally, consider the steady state effects of government expenditure and taxes for the above model (*table 6.5*). These results refer again to the stable (Christ) regime $\gamma_1=0;\gamma_2=1$. The change in g_o is 1 per cent of total income y, and the changes in the tax rates are such that they produce an equivalent impact effect on the government's budget. As might be expected the rise in government expenditure leads to a higher interest rate and larger public debt. In order to maintain the rate of growth the profit rate must rise as well. Because of the higher interest rate the corporate debt ratio declines a little.

Table 6.5 Steady state effects of a stimulus of 1% of y for different fiscal instruments

instrument	effect on b	m	a	π[1])	r[1])
g_o	0.152	-0.002	-0.009	0.28	0.56
τ_o	0.151	-0.001	-0.009	0.31	0.53
τ_1	0.176	0.001	-0.001	0.05	0.08
τ_2	0.153	-0.001	-0.020	-0.08	0.04

[1]) effects in percentage-points.

The ultimate increase in government expenditure is less than the initial increase, because the larger debt service leads to some internal crowding out of expenditure (because $\gamma_2=1$). Furthermore, the final increase of government expenditure occurs at the expense of private consumption. This is accomplished by the higher interest rate and the shift in income distribution in favour of profits. In this model with a fixed technology and a given rate of growth there is no long-term crowding out of investment. There will, of course, be some crowding out in the short run through the higher interest rate, but eventually this negative effect on investment will be offset by a higher profit rate. It is obvious that if the technology is endogenous, the higher profit rate will lead to a decline in the desired capital-output ratio, and thus to some crowding out of investment too. This effect will not, however, qualitatively change our results above.

The steady state effects of a reduction in taxes on wages (τ_0) are practically identical to those of the rise in g_0. A reduction in tax on the interest income of workers (τ_1) has smaller effects on the profit rate and the interest rate, but gives rise to a sharper increase in public debt. Obviously, the reduction in τ_1 not only raises the budget deficit directly, but also weakens the mitigating feedback of rising interest payments on tax receipts, and thereby on the growth of public debt. In contrast with the other cases, a reduction in taxes on net corporate returns (τ_2) leads to a fall in the profit rate. This is because less profits (before taxes) are necessary to keep up growth when taxes are lower. The steady state effects on public debt, the money stock and the interest rate are basically similar to those of the other fiscal policy instruments.

6.5 CONCLUSION

In this chapter we have investigated the dynamics of growth and asset accumulation from a long-term point of view. With regard to income-expenditure equilibrium it was assumed that the monetary authorities have sufficient time to find the right policy to maintain full utilization of capacity. Therefore demand dynamics is shifted into the background; instead the interaction between asset accumulation and income distribution emerged as the principal mover of economic development. For a plausible numerical model we have established that the stability directly depends on the natural rate of growth; the lower this growth rate the less stable is the system. Beyond some minimum growth rate (the bifurcation point of the catastrophe manifold) the system loses its stable solution altogether. Just as in the medium term the system proved to be more stable in an upward than rather than in a downward direction.

The fiscal policy regime proves to have a great impact on the system's stability, especially the budgetary reaction to changes in nominal interest payments (γ_2). The rigid Blinder-Solow regime with γ_2=0 emerges as the least stable regime, and the Barro regime, which adopts a target for a constant debt ratio, as the most stable. The scheme below gives a comparison of the ranking of the regimes for the medium period and the long period. These rankings bring out that it can be concluded that, although the Barro regime seems best from a long-term point of view, it performs worst in the medium period. The Blinder-Solow is the least stable from a long-term point of view, and after the Barro regime, the least stable in the medium period as well. A more attractive regime seems to be the Christ regime which performs best in the medium period and, after the Barro regime, best in the long period as well, closely followed by the Tobin-Buiter regime. Each of these regimes implies automatic reduction of government expenditure when interest payments increase. In the medium term this 'internal' crowding out of government expenditure produces an anti-cyclical

	medium period	long period
stable	Christ	Barro
|	Tobin-Buiter	Christ
|	Blinder-Solow	Domar
\/	Domar	Tobin-Buiter
unstable	Barro	Blinder-Solow

movement of expenditure; in the long term it mitigates the cumulative spiral of increasing debt and interest payments.

Note that whether a regime leads to a stable system or not depends on the model as a whole, and especially on the natural rate of growth. Therefore, any structural change in the economy may turn a stable system into an unstable one. Furthermore, it is important to note that large shocks, especially downward disturbances, may lead to a displacement of the system from a 'stable' region, from where it will always return to its steady state, into an 'unstable' region, from where it recedes further and further from steady state equilibrium. These observations lead to the conclusion that there is no automatic rule or regime for fiscal policy which guarantees stability under all circumstances. Rules need to be reconsidered regularly.

CHAPTER 7

STABILITY OF PUBLIC DEBT IN AN OPEN AND GROWING ECONOMY

7.1 INTRODUCTION

In this chapter we shall generalize the foregoing analysis for the open economy.[1] It is remarkable that research on the dynamics implied by the government budget constraint is generally restricted to the closed economy. This concerns the IS/LM oriented studies in the tradition of Blinder and Solow (1973) as well as earlier studies on the role of public debt in growth models (cf. Domar 1957, Modigliani 1961, Diamond 1965).[2] Notable exceptions are the contributions of Turnovsky (1976) and Nguyen and Turnovsky (1979). In this chapter it will be shown that the dynamics of growth and asset accumulation is essentially different from those of the closed economy. In order bring out these differences as clearly as possible, we shall concentrate on the extreme case of a small open economy, which is a price-taker on the international commodity and financial markets.

The basic ingredients of the present analysis are the following: First, as mentioned, we focus on the open economy. The government budget constraint (GBC) is treated in the tradition of the IS/LM based studies mentioned above, allowing for bond financing as well as for money financing. The modelling of the relationship between the balance of payments constraint (hereafter abbreviated as BPC) is akin to that of the small open-economy models of Domar (1957), Hamada (1966) and Neher (1970), but with two important modifications. First, in order to bring out the specific role of saving from interest income the present analysis assumes a 'Kaldor' differential savings function. Secondly, the domestic interest rate is not determined by the international interest rate alone, but is dependent on the net international creditor or debtor position as well. Finally, the present analysis takes account of the dynamic relationship between asset accumulation and income distribution by incorporating a simple 'conflict' model of income distribution.

The chapter is organized as follows. After some preliminary considerations on the determination of growth and income distribution in an open economy in section 7.2,

[1] This chapter is largely based the article "Interest payments and the stability of the government budget deficit in an open and growing economy" in De Economist (Van Ewijk 1986a). An earlier and more extensive version of this article was circulated as a research memorandum (Van Ewijk 1985).

[2] The neglect of the open economy aspects may be related to the fact that research on the stability of public debt is geographically concentrated in the United States,

we shall develop the 'static' part of the model describing the instantaneous equilibrium relationships which are required to hold continuously (section 7.3). Then, sections 7.4-7.6 analyse the dynamics of the model which arises from the GBC, the BPC and the interaction between growth and income distribution. Section 7.7 deals with the characteristics of the steady state equilibrium. Finally, section 7.8 examines the consequences when it is assumed that the government adopts a target for the balance of payments on current account instead of some fixed budgetary target.

7.2 GROWTH AND DISTRIBUTION IN THE OPEN ECONOMY

Before developing the open economy model we first have to discuss some problems which arise when the post-Keynesian model, which is typically designed for the closed economy,[3] is applied to the open economy. One of the basic theorems of post-Keynesian theory, as we have discussed it in chapter 2, is that income distribution is solved from the side of demand for output. Assuming a differential propensity to save from wages and profits, it is argued that discrepancies between aggregate demand and supply are solved by shifts in income distribution. As has been discussed in chapter 2, the adjustment in income distribution is achieved by changes in the price level while nominal wages are relatively rigid.

In this theory the nominal wage rate plays only a minor role. Apart from the special situation in which the economy is driven towards the inflation barrier (see chapter 2), nominal wages function primarily as the numeraire, fixing the general level of prices. In an open economy this 'neutrality' view of nominal wages is no longer warranted. In the presence of international competition on the commodity markets a change in nominal wages affects the competitive position of domestic firms vis-a-vis foreign competitors. As a consequence changes in money wages cannot simply be passed on in prices without affecting the competitive strength of domestic firms. In this environment money wages may therefore have a decisive influence on real wages and the profit rate.

This has important consequences for the standard post-Keynesian model. International competition on goods markets seriously impedes the possibility of restoring equilibrium between aggregate demand and supply through the redistribution of income. In the extreme case of full arbitrage on the goods markets (law of one price) there is no room for this redistribution mechanism at all: the profit margin and real wages are completely determined by domestic nominal wages and the

[3] See e.g. Robinson (1956, 1962), Kaldor (1956,1961), Pasinetti (1974), Kregel (1975) and Harris (1978). It is illuminating that Harris (1978, p.VIII) when summing up the omissions of his textbook does not even mention the total neglect of the international aspects of growth! Some contributions on particular aspects of the international theory of growth and distribution have been made by Kregel (1975), Steedman (1979), Steedman and Metcalfe (1979) and Pasinetti (1981).

international price level at the given exchange rate. The well-known post-Keynesian causation from investment to prices and income distribution breaks down in this case.[4]

In principle the effect of a change in money wages may be offset by an equivalent change in the exchange rate. This, however, requires the monetary authorities to accommodate passively any change in the domestic price-level. As this is not very plausible, we shall leave this possibility out of consideration. This does not necessarily imply that the exchange rate must be fixed. It may vary, but it is independent of (changes in) the domestic level of wages.

Now imagine a rise in aggregate demand, caused, for example, by an increase in investment. In the case of a substantial price elasticity of the foreign supply of goods, the rise in demand leads to a increase in imports - when domestic capacity is fully utilized - rather than a rise in domestic prices. Hence, the link between demand and income distribution is now replaced by a link between demand and the balance of trade. Thus in the long period[5] discrepancies between ex ante savings and investment are not resolved by redistribution of income as in the closed economy, but are reflected in the balance of payments on current account.

The post-Keynesian position on these matters is not very clear. For the long term it is generally recognized that foreign prices exert a significant influence on domestic prices; however, on the strength of this influence opinions appear to diverge. 'Ricardian' post-Keynesian authors such as Steedman and Metcalfe (1979) and Pasinetti (1981) tend to accept that prices are fully determined by international prices. More 'Keynesian' post-Keynesians, as for example Kregel, hold on to the proposition that, although foreign prices may indeed have some influence, domestic factors still have a significant impact.

International equalization of profit rates

Now consider the determination of the profit rate. As we have seen, post-Keynesian theory explains the profit rate in a closed economy by the savings propensities and the rate of growth. It needs little argumentation to see that in the presence of international capital mobility this explanation can no longer be sustained. On the contrary, it may be expected that competition on international financial markets produces a tendency towards the international equalization of profit rates.

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[4] Robinson (1962, p.70) mentions that foreign competition may raise problems with respect to the distribution mechanism, but gives it no further consideration. Kregel (1975, ch.12) pays more attention to this subject, recognizing that foreign competition 'may limit the ability of domestic producers to set the prices they desire,' but fails to give a clear account of the long-term implications of the openness of the economy for the determination of income distribution.

[5] Kalecki (1934), however, analyses a similar link between the balance of trade and demand, and therefore profits with a reverse causation. In his fixed-price short-term model the balance of payment on current account provides an additional (external) source of demand besides investment and government expenditure. In the long period, however, when capacity is fully utilized, the causation runs from demand to the balance of trade.

Overviewing post-Keynesian theory it appears that, although most authors recognize the tendency towards equal profit rates,[6] it is generally treated in a ambivalent manner. For example, Steedman (1979) recognizes that capital mobility would imply equalization of profit rates, but subsequently excludes this possibility by postulating strict immobility of capital. Steedman and Metcalfe (1979) do allow the money rate of interest to be determined internationally, but relate the profit rate exclusively to domestic factors; the interest rate only functions as a minimum below which the profit rate cannot fall. Also Pasinetti (1981) is ambiguous on this point; while he explains sectoral profit rates in a traditional post-Keynesian fashion by the (sectoral) rates of expansion, he nevertheless assumes profit rates to be equal internationally too. However, this is not explained by international capital mobility, but follows from Pasinetti's assumption "for simplicity" that growth rates for each sector are equal in every country (Pasinetti 1981, p.246).[7]

In the model to be developed in this chapter we shall adopt the Ricardian position and assume complete price arbitrage on commodity markets. With respect to the equalization of profit rates we shall develop a more dynamic view which allows for international differences in real interest rates and profit rates, despite full international mobility of capital.

7.3 THE MODEL

In order to be able to concentrate on the basic 'laws' governing the accumulation of financial assets the real side of the economy is represented by an elementary model of steady growth. It is assumed that capacity is always fully utilized, that technical change is purely labour augmenting and that the capital-output ratio is constant. Hence the growth rates of production and employment (in efficiency units) are equal to the growth rate of capital stock (i). The discrepancy between this rate and the (exogenous) growth rate of labour supply (the 'natural' rate of growth) determines the evolution of unemployment. Further, we shall adopt the small open-economy framework, which implies that the economy is a full price-taker on both the international financial and the commodity markets.

In order to reduce the dimensionality of the model in this chapter we shall not distinguish between the corporate sector and workers, but adopt a Kaldorian

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[6] Note that Kregel (1975) does not pay any attention at all to this tendency.

[7] A third problem with respect to the post-Keynesian for the open economy concerns the determination of relative prices. If one follows Sraffa and assumes that relative prices are determined by income distribution and fixed technical coefficients, international competition on goods markets will lead to full specialization of production in each country. In macroeconomic models this is often neglected by adopting a one-sector model (cf. Steedman and Metcalfe 1979). Pasinetti solves this problem by introducing decreasing returns to scale on the sector level. In Van Ewijk (1983) we discussed these matters more in detail. In the following analysis we shall avoid this problem by assuming homogeneity of goods.

consumption function which distinguishes between wages and property income. If it is taken into account that the property income of the private sector consists of profits from domestic production as well as real interest income received on financial wealth,[8] the income identities of the model can be written as follows (all stocks and flows are expressed in ratios to capital stock):

$$y = w + \pi \tag{7.1}$$

$$y_2 = \pi + rb + re - pm \tag{7.2}$$

where y = domestic production
w = wage income
π = profit rate
y_2 = private property income
r = real interest rate
b = government debt
e = net external creditor (+) or debtor (-) position
m = (base) money

The first equation divides domestic production between wages and profits. The second one defines private property income as the sum of profits and interest income; it has been assumed that no nominal interest is paid on money.

The expenditure relations are straightforward. In order to bring out the specific role of spending from property income the consumption function distinguishes between the propensity to consume from disposable wage income c_1 and from disposable property income c_2. This simple Kaldorian function is modified by including a wealth effect (c_z) and an interest effect (c_r) on consumption as well.[9]

$$c = c\{(1-\tau_1)w,(1-\tau_2)y_2,r,z\} \qquad c_1,c_2,c_z \geq 0;\ c_r \leq 0 \tag{7.3}$$

where z = $1 + e + b + m$ (= total private wealth as a ratio to capital stock (=1))
c = consumption
τ_1 = tax-rate on wages
τ_2 = tax-rate on property income

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[8] These latter components of private sector income have been neglected in most earlier studies on public debt. Cf. Domar (1957), Diamond (1965) and Christ (1968).

[9] It has been assumed that consumers are not ultra-rational in the sense put forward by Barro (1974) and others, that they fully capitalize the effect of current issues of public debt on future taxes (see for a critique of this so-called 'Ricardian doctrine', Tobin 1980).

and c_1 and c_2 denote the first order derivatives of c with respect to net wage income and net property income respectively. With regard to taxes no distinction is made between the different components of property income. This implies that inflationary losses on nominal assets are taxed at the same rate as nominal interest income. Although this is not wholly realistic, it does not seem an unreasonable simplification at the present level of abstraction.

In a price-taking economy the rate of investment, and thus the growth rate of production should be related to the profit rate π, the interest rate r and the tax-rate on profits τ_2 and a term ζ representing all other (psychological) factors like the optimism and the readiness to take risks, or, in Keynes' words, the 'animal spirits' of entrepreneurs.

$$i = i(\pi, r, \tau_2, \zeta) \qquad i_\pi, i_\zeta > 0;\ i_r, i_\tau < 0 \tag{7.4}$$

As regards the budgetary regime we shall for the moment follow Blinder and Solow (1973) and take government expenditure (g) and tax rates (τ_1, τ_2) to be fixed. Later we shall consider other regimes as well.

Then with given c, i and g and it can be assessed that the balance of trade (f) must be equal to the gap between domestic production (y) and aggregate expenditure,

$$f = y - g - c - i \tag{7.5}$$

where f = balance of trade. This equilibrium will be ensured by the fact that in a small open economy with perfect competition on international markets, domestic producers will be able to sell all goods produced at capacity level at the prevailing international prices.

The financial sector

So much for the real part of the model. For the financial sector straightforward relations will be chosen. The small-open-economy assumption entails that prices always satisfy purchasing-power parity. The nominal interest rate (R) is assumed to be equal to the international interest rate (R_f) plus the (expected = actual) rate of depreciation $(D\Theta/\Theta)$ plus a factor (σ) measuring the risk premium on domestic assets for international portfolio-holders. Following a suggestion by Hamada (1969, p. 684) this risk premium is assumed to be related negatively to the foreign creditor position (e), or equivalently, positively to the volume of foreign debt. All other factors determining σ are assumed to be constant in the present analysis. If it is finally assumed that demand for money (M) is a simple homogeneously linear function of domestic production (Py), the financial sphere can be modelled as:

$$P = \Theta.P_f$$
$$R = R_f + D\Theta/\Theta + \sigma(e) \qquad \sigma'<0$$
$$M = m.Py \qquad m = \text{constant}$$

where P = price level
Θ = exchange rate
R = nominal interest rate
σ= (risk)premium on domestic currency
M= money stock

The suffix f denotes foreign variables. It is well-known that these equations reduce to the following relations in real terms:

$$r = r_f + \sigma(e) \tag{7.6}$$

$$DM/M = p + i \tag{7.7}$$

where r_f is the foreign real interest rate. This result leads to the familiar conclusion in these kind of models that inflation is a purely monetary phenomenon and that the real interest rate is given by real factors. As to the regime of monetary policy the government can choose for either fixing the growth of money stock or adopting a fixed target for the rate of inflation (and thus depreciation) and adjusting money growth to that target. The difference between those regimes may be important when the growth rate of production varies. In what follows it will be assumed that the government chooses the latter option, so that monetary policy is adequately represented by the target for the rate of inflation.

Now the static part of the model is completed. Its structure is very simple. It consists of 7 independent equations, 7 endogenous variables (w, y_2, r, c, i, f, DM/M), 4 policy variables (g, τ_2, τ_1, p) and 3 state variables (e, b, π) which are historically given and will be explained in the next section.

7.4. THE DYNAMICS

The dynamics of the model arises from the GBC and the BPC and the interaction between growth and income distribution. With respect to this latter process, it will be assumed that the rate of profit is determined by a conflict model of income distribution, the common outcome of which is that (in the medium term) the rate of change in π depends in one way or another on the evolution of the unemployment rate. Since the change in the unemployment rate in turn depends on the difference between the actual and the natural growth rate, in our model this relationship may, as in the foregoing chapter (eq. 6.7), captured in the following linear expression:

$$D\pi = \vartheta_\pi(n - i) \qquad \vartheta_\pi>0 \tag{7.8}$$

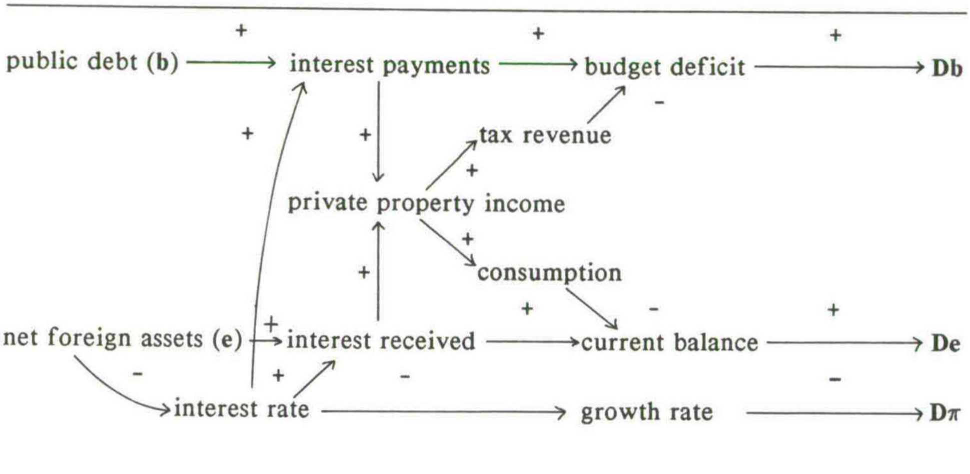
where n = growth of labour supply (natural rate of growth).

The evolution of the financial stocks e and b depends on the GBC and BPC. If one takes into account that the ratios of money and debt are continuously being reduced by the rate of inflation and the growth rate of production, these budget constraints are given by

$$\text{BPC:} \quad De = y - g - c - i + (r+p)e - (i+p)e \tag{7.9}$$

$$\text{GBC:} \quad Db+Dm = g + (r+p)b - \tau_1(y-\pi) - \tau_2(\pi+rb+re-pm) - (i+p)(b+m) \tag{7.10}$$

where r+p = nominal interest rate. According to the first equation the change in the foreign lending position depends on the balance of payments on current account (=f+(r+p)e) less the reduction in e through nominal production growth (i+p)[10]. The second equation states that the change in public debt and money stock is equal to government outlays (including interest payments) less tax revenue and less the reduction in b and m through nominal production growth. Since the ratio of money to national product is assumed to be constant (hence Dm=0) the fundamental causal relationships resulting from the GBC and the BPC can be illustrated by the following scheme:



[10] It has been assumed that the net stock of foreign reserves is nil, so that the balance of payments on current account must be fully financed by interest bearing debt.

This scheme, in which for expository reasons the effects of π have been disregarded, shows that greater public debt leads to greater interest payments to the private sector and thus on the one hand to a larger budget deficit and on the other hand to larger private property income. This larger property income increases tax receipts, thus mitigating the increase in the government's budget deficit, and consumption, thereby worsening the balance of payments on current account. This is one line in the scheme. The other runs from the net foreign position (e) to interest received from abroad and thus to the current account and via property income to tax revenue and the budget deficit. In addition e influences the system through its impact on the interest rate and thus on interest payments and the growth rate.

The impact of the profit rate on Db and De - not shown in the scheme - is a rather complex one. In the first place a higher π leads to a higher growth rate and thus to a reduction in the absolute magnitude of Db and De. On the other hand a higher growth rate induces larger investments and therefore a decline of the trade balance. Finally, a change in π will change income distribution and thus tax revenue and consumption. Under the plausible assumptions that $\tau_2>\tau_1$ and $c_2<c_1$ a higher profit implies higher tax revenue (thereby reducing Db) and lower consumption (thereby increasing De).

Local stability

Whether or not this system is (locally) stable around its steady state solution (e_s, b_s, π_s) can be established from the linearized system based on equations (7.8), (7.9) and (7.10).

$$\begin{bmatrix} De \\ Db \\ D\pi \end{bmatrix} = \begin{bmatrix} h_1-n & -h_2 & -h_5 \\ -h_3 & h_4-n & -h_6 \\ -\vartheta_\pi i_r \sigma' & 0 & -\vartheta_\pi i_\pi \end{bmatrix} \cdot \begin{bmatrix} e-e_s \\ b-b_s \\ \pi-\pi_s \end{bmatrix} \tag{7.11}$$

where $h_1 = r - c_2(1-\tau_2)r - c_z - \sigma'\{c_r+c_2(1-\tau_2)(b+e)-i_r(1+e)-e\}$

$h_2 = c_2(1-\tau_2)r + c_z$

$h_3 = \tau_2 r + \sigma'\{\tau_2(b+e)+i_r(b+m)-b\}$

$h_4 = (1-\tau_2)r$

$h_5 = i_\pi(1+e) - \{c_1(1-\tau_1)-c_2(1-\tau_2)\}$

$h_6 = i_\pi(b+m) + (\tau_2-\tau_1)$

This system is stable if the real parts of the characteristic roots of the state matrix (subsequently denoted as the **H** matrix) are negative, or by the modified Routh-Hurwitz conditions if

I. Trace (H) < 0 (7.12)

II. Det (H) > 0

III. $\text{Det}\begin{bmatrix} 2n-h_1-h_4 & h_6 & -h_5 \\ 0 & n-h_1+\vartheta_\pi i_\pi & h_2 \\ -\vartheta_\pi i_r \sigma' & h_3 & n-h_4+\vartheta_\pi i_\pi \end{bmatrix} > 0$

These conditions are difficult to deal with on an analytical level. Therefore we will start with a somewhat simpler specific case in which the real interest rate is independent of the net foreign debt, $\sigma'=0$. In this manner we will be able to trace some of the basic determinants of stability. Thereafter we will return to the general model and check whether the characteristics found for the specific case are also valid if the assumption $\sigma'=0$ is dropped.

7.5 A SPECIAL CASE: EXOGENOUS INTEREST RATE

When $\sigma'=0$ the real interest rate is simply given by the international interest rate and thus exogenous to the present model. This considerably simplifies the solution of the conditions for stability. From equations (7.11) and (7.12) it can be found that the Routh-Hurwitz conditions now reduce to

I. $2n - h_1 - h_4 + \vartheta_\pi i_\pi > 0$ (7.13a)

II. $\vartheta_\pi i_\pi\{(n-h_1)(n-h_4) - h_2 h_3\} > 0$

III. $(2n-h_1-h_4)\{(n-h_1)(n-h_4) - h_2 h_3 + \vartheta_\pi i_\pi(2n-h_1-h_4-\vartheta_\pi i_\pi)\} > 0.$

If both conditions (I) and (II) are satisfied, it can easily be seen that the third condition can only be fulfilled if $\vartheta_\pi i_\pi>0$ and $(2n-h_1-h_4)>0$. This implies that the above conditions can be reduced to

I. $2n - h_1 - h_4 > 0$ (7.13b)

II. $(n-h_1)(n-h_4) - h_2 h_3 > 0$

III. $\vartheta_\pi i_\pi > 0$

This result is interesting for it implies a kind of dichotomy in the stability conditions. That is, these conditions would also have been found if the dynamics of growth and income distribution on the one hand and the accumulation of financial assets on the other hand were considered separately. Condition (III) is conclusive for the stability of income distribution and growth, and given a certain growth rate the conditions (I) and (II) determine the stability b and e.

Conditions (I) and (II) can be reduced by substituting for the h's

$$n - r > 0 \tag{7.14}$$

$$n - (1-c_2)(1-\tau_2)r + c_z > 0$$

Since $0 \leq c_2 \leq 1$, $0 \leq \tau_2 \leq 1$ and $c_z \geq 0$ it can easily be seen that the first condition is conclusive to stability for any $r \geq 0$. This is a very simple and clear-cut result. Since the interest rate is given internationally it means that the domestic growth rate n is the principal determinant of stability. Slowly growing economies will therefore be much more liable to unstable asset accumulation than similar economies with rapid growth. In a stationary economy (n=0) instability proves even to be inevitable, unless the real interest rate is negative. This confirms the pessimistic view on stability emerging from studies based on stationary IS-LM models, as mentioned in chapter 6. In our model this proves even to be true irrespective of the wealth elasticity of consumption c_z.

In addition to this main point we should make two other observations. First, the stability of the system is *independent* of the rate of inflation, and thus of the rate of depreciation. This implies that monetary policy cannot - in the long run - alter the stability of debt accumulation. This contrasts with the conclusion of Blinder and Solow and many others that the model will be less unstable as the degree of money financing is greater. In the present model even tax policy is ineffective with respect to stability or instability. This does not mean, however, that the monetary and fiscal policy are not important at all. Below we shall see that they may have some impact on stability when other policy regimes are chosen. Furthermore these policy instruments will be shown to have an important impact on the steady state values of b and e (section 7.4).

Secondly, it must be pointed out that for other fiscal policy regimes less stringent conditions for stability are found. So far we have followed Blinder and Solow in taking government expenditure (g) as the relevant exogenous policy variable. As we have discussed in chapter 5, several alternative regimes have been put forward in the literature, viz.

a. fixed expenditure including interest payments net of taxes (Tobin-Buiter);
b. fixed expenditure including gross interest payments (Christ).
c. fixed budget deficit (Domar);
d. balanced budget (Buchanan);
e. fixed debt ratio (Barro)

The results for these policy regimes are presented in *table 7.1* below. Since p cannot be less than -r (otherwise the nominal interest rate would be negative) the conditions for each alternative regime prove to be less stringent than in the original case with fixed g. This is obviously due to the fact that these regimes imply g to decline endogenously as interest payments rise. It should however be noted that this automatic 'crowding out' is bounded by the condition that government expenditure should not become negative.

Table 7.1 Conditions for stability under alternative policy regimes*

regimes		conditions	
a. Blinder-Solow	: fixed g	n-r>0	$n-(1-c_2)(1-\tau_2)r+c_z>0$
b. Tobin-Buiter	: fixed $g' = g + (1-\tau_2)(r+p)b$	$n-r + (1-\tau_2)(r+p)>0$	$n-(1-c_2)(1-\tau_2)r+c_z>0$
c. Christ	: fixed $g'' = g + (r+p)b$	n+p>0	$n-(1-c_2)(1-\tau_2)r+c_z>0$
d. Domar	: fixed deficit	n+p>0	$n-(1-c_2)(1-\tau_2)r+c_z>0$
e. Buchanan	: zero deficit		$n-(1-c_2)(1-\tau_2)r+c_z>0$
f. Barro	: fixed b		$n-(1-c_2)(1-\tau_2)r+c_z>0$

* In all regimes it is assumed that the target is achieved by adjusting g. A proof of these results is given in *Appendix 7.A*.

These results show that under each regime the condition $n-(1-c_2)(1-\tau_2)r+c_z>0$ is a necessary, though not always a sufficient condition for stability. It is evident that whenever this condition is conclusive, the government has a powerful instrument for ensuring the system's stability, namely the tax rate on property income τ_2. The conditions in the first column turn out to be different for each regime. For the Domar, Tobin-Buiter and Christ regimes, they imply that monetary policy may contribute to stability by ensuring that the inflation rate is not too low ($p>-n$ in regime c. and d., and $p>(\tau_2 r-n)/(1-\tau_2)$ in b.). Thus unlike the reference regime these alternative regimes leave considerable room for fiscal and monetary policy to influence the stability of the system.

Another interesting feature of the alternative regimes is that the system may be stable even in the absence of real growth. Provided that $p>0$ (in regime c. and d.) or $p>\tau_2 r/(1-\tau_2)$ (in regime b.) the evolution of debt will be stable if the wealth elasticity of consumption is sufficiently large; that is, if $c_z>(1-c_2)(1-\tau_2)r$. This corroborates the basic conclusion of the stationary IS/LM models that a positive wealth effect is an essential prerequisite for stability of public debt, but is, of course, in contrast with the conclusions of the foregoing chapter.

7.6 THE GENERAL CASE

Returning to the general case it can first be shown that the conclusion on the decisive role of the growth rate and the interest rate is also valid when $\sigma'<0$. Solving the

Routh-Hurwitz conditions (7.12) it is found that

I. $2n-h_1-h_4+\vartheta_\pi i_\pi>0$

II. $\vartheta_\pi i_\pi\{n-h_1)(n-h_4)-h_2h_3\} + \vartheta_\pi i_r\sigma'\{h_2h_6-(n-h_4)h_5\} > 0$

III. $(2n-h_1-h_4)(n-h_1)\{(n-h_4)-h_2h_3 + \vartheta_\pi i_\pi(2n-h_1-h_4+\vartheta_\pi i_\pi)\}$
$$- \vartheta_\pi i_r\sigma'\{h_2h_6+(n-h_1)h_5+\vartheta_\pi i_\pi h_5\} > 0$$

Provided that $\vartheta_\pi i_\pi>0$ it can be seen that condition I is a positive linear function of n, condition II a positive quadratic function of n and condition III a cubic function of n with a positive first term. This implies that the system will always be stable whenever the growth rate is sufficiently high. In a similar way it can be established that stability is also ensured if the real interest is sufficiently low.[11]

Since these conditions are difficult to handle in more detail on the analytical level it may be useful to present some numerical results for plausible values of the parameters. A convenient device for assessing the stability characteristics of the model is to calculate the growth rate that is at the minimum required for stability, n_{min} (see section 6.3). Then, the lower this minimum growth rate is the more unlikely it will be for the system to be unstable.

Figure 7.1 presents the results for n_{min} in relation to the net creditor position e for the case with zero wealth elasticity and for the case with high wealth elasticity (c_z=0.1). For all growth rates in the area above these curves the system is stable.
In both cases the Blinder-Solow regime with fixed g proves again to be significantly more restrictive than the other regimes, especially when the wealth elasticity is high. Further these figures bring out that the system in general becomes more stable if e/y is larger. This means that a country with large foreign debts will be more liable to instability than a net creditor country. This general rule is, however, violated for a certain interval with high v's under the Tobin-Buiter regime (fig. a).

Further it may be noticed that in *fig. b* the Domar regime yields Domar's original condition for stability, namely that the nominal rate of growth should be positive (hence n>-p and thus n>-4%). Finally, a remarkable result of the present analysis is that for the Blinder-Solow regime a high wealth elasticity of consumption turns out

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[11] Denoting the Routh-Hurwitz conditions by RH1, RH2 and RH3 it is obtained by differentiation that

$dRH1/dr = - 1 - (1-c_2)(1-\tau_2) < 0$

$d^2RH2/dr^2 = 2\vartheta_\pi i_\pi(1-c_2)(1-\tau_2) > 0$

$d^3RH3/dr^3 = - 6(1-c_2)(1-\tau_2)\{1-(1-c_2)(1-\tau_2)\} < 0$

These results imply that RH1 is a negative linear function of r, RH2 a positive quadratic function of r and RH3 a cubic function of r with a negative first term. These characteristics ensure that all conditions will be satisfied for sufficiently low (possibly negative) values of r.

Figure 7.1 Minimum growth rate required for stability

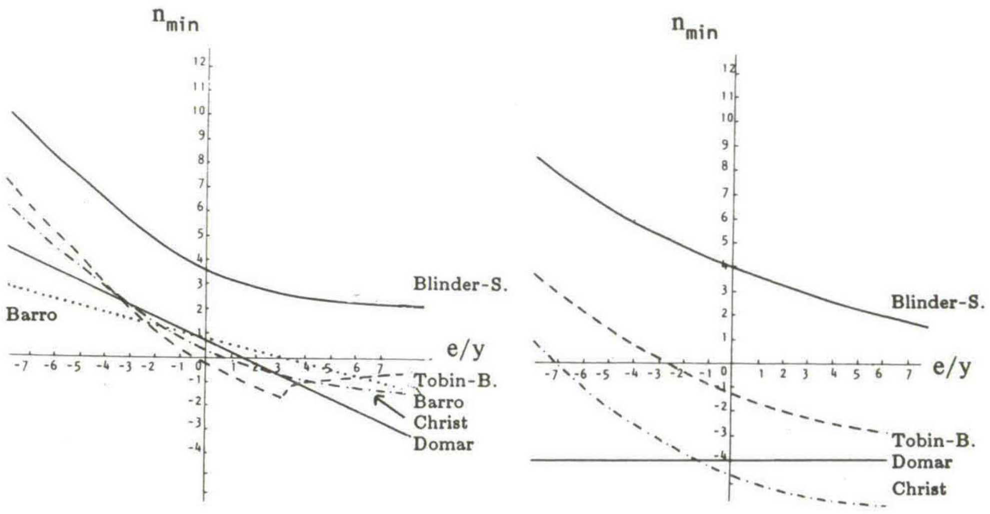


fig. a. (c_z=0) fig. b. (c_z=0.1)

Explanation: This figure is based on the following parameter values: y=0.5; c_2=0.2; c_1=0.9; τ_2=0.4; τ_1=0.3; c_r=-0.2; σ'=-0.01; i_π=0.5; i_r=-0.5; m/y=0.1; b/y=0.5; ϑ_π=0.25; p=0.04; r=0.04. The results for the Buchanan regime are similar to those for the Domar regime. For the Barro regime with c_z=.1 (fig. b) the critical growth rate varies from -7.1% if e/y=-7 to -10.5% for e/y=7.

to make the system less stable. This confirms the conclusion of the foregoing chapter (as well as chapter 2). For other regimes, however, our results seem to confirm the usual findings.

How the minimum growth rate depends on the other variables can be seen from *table 7.2*, which presents the partial derivatives of n_{min} for the reference regime at three alternative levels of e in relation to domestic production y. The signs of most coefficients conform with what might intuitively be expected. Yet several observations may be made. First, it is important to note that a change in r leads to a practically one-for-one change in n_{min}. Secondly, unlike in the case with a purely exogenous interest rate, fiscal policy now appears to have some influence on the system's stability through the tax rates τ_2 and τ_1. However, the coefficients are very low. If, for example, τ_2 is raised by 10 percentage points the minimum growth rate falls by only 0.24 points (if e=0). Monetary policy turns out to be totally ineffective again. Finally

Table 7.1 Partial derivatives of n_{min}

	e/y=-2	e/y=0	e/y=2
r	0.993	0.968	0.939
p	0	0	0
e/y	-0.005	-0.004	-0.004
b/y	0.005	0.005	0.005
m/y	0	0	0
τ_1	0.019	0.018	0.017
τ_2	-0.005	-0.024	-0.042
c_z	0.004	0.012	0.011
ϑ_π	0	0	0

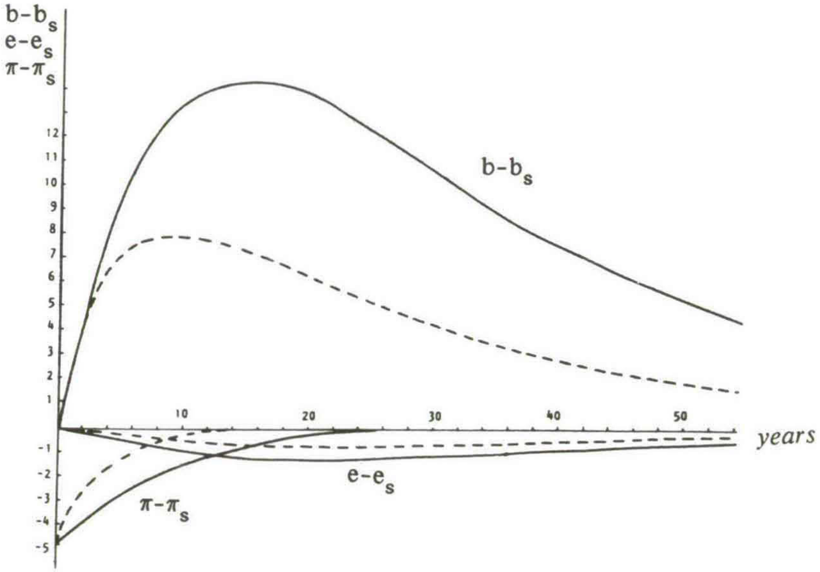
Explanation: The parameter values are the same as in figure 7.1b.

these results confirm our observations from *figure 7.1* that n_{min} goes down as e increases (and b decreases) and that a higher wealth elasticity pushes n_{min} up.

A somewhat remarkable result is that the speed of adjustment of the profit rate (ϑ_π) does not seem to influence n_{min}. In our numerical example this is due to the fact that the second Routh-Hurwitz condition, which is independent of ϑ_π, is conclusive to stability. Only when ϑ_π becomes very small does it appear to have an impact on n_{min}. This suggests that the interaction between growth and income distribution is a relatively 'fast' process in comparison to the dynamics of debt accumulation. This does not at all mean, however, that ϑ_π is unimportant; it can be shown to have an important impact on the shape and the duration of the adjustment trajectory. This is demonstrated in *figure 7.2*, which is based on a simulation of the adjustment process starting from an initial situation characterized by a profit rate 5 percent points below its steady state value π_s.

This figure brings out that the profit rate adjusts fairly rapidly to its steady state value. However, because during the first period the growth rate is below its equilibrium value government debt moves away from its steady state level reaching a peak of nearly 15 per cent (of domestic production) above b_s. That the accumulation of net foreign assets is much less affected, is due to the fact that the lower profit rate and the growth rate have opposite effects on the balance of payments. Whereas the lower profit rate tends to create a deficit on current account because it increases consumption, the lower growth rate mitigates this effect because investment will be less. With the given parameter values, the first effect proves to be somewhat stronger so that the lower profit rate leads to a modest worsening of the international debtor position. When π approaches its steady state level the growth of public debt and foreign debt levels off, whereafter a long phase of steady decline of b and e sets in.

Figure 7.2 The adjustment trajectory



Explanation: This figure is based on a dynamic simulation of the linearized system (equation 7.11) starting from $e-e_s=0$, $b-b_s=0$ and $\pi-\pi_s=-0.05$. The natural growth rate is 6% and the interest rate 2%. All other variables have the same values as in figure 7.1a.

In the case of a higher speed of adjustment of π ($\vartheta_\pi=0.5$) the profit rate and the growth rate recover much faster, so that the amplitude of the movements in s and e are less wide.

7.7 STEADY STATE EQUILIBRIUM

Along the steady growth path all financial stocks must grow by the same rate as domestic product. As the growth of assets must be provided for by the government budget deficit and the surplus in the balance of payments on current account, these accounts will in general not be in balance. In nominal terms these accounts must satisfy

$$\begin{aligned}\text{budget deficit} &= DB + DM = (n+p)(m+b)PK\\ \text{current account} &= DE = (n+p)ePK\end{aligned}$$

Where PK is the nominal capital stock, and B, M and E the nominal stocks (in absolute terms) of public debt, money and foreign assets.

Christ (1979) has pointed out for the closed economy that in the stationary fix-price models of Blinder-Solow and Tobin-Buiter equilibrium must be characterized by a balanced budget. But if one allows for inflation as in Christ (1978, 1979) this result is modified since then *real* stocks of financial assets need to be constant. This requires the government budget to show a deficit in order to satisfy the growing need for nominal assets due to inflation. It will be evident that in a growing economy the GBC must satisfy the increase in demand for nominal assets due to inflation as well as to real growth.

For an open economy the same reasoning can be applied to the BPC. In a zero growth, zero inflation economy (cf. Turnovsky 1976), the current account must be balanced. In a zero growth economy with inflation (cf. Nguyen and Turnovsky 1979, 1983), the current account must show a surplus when $e>0$ or a deficit when $e<0$ to supply the growing need of foreign assets or liabilities due to inflation, while in a growing economy it must supply the need due to real growth as well.

How the financial stocks and income distribution are influenced by the other variables in this model can be established from the total differential of equations (7.8), (7.9) and (7.10) subject to the steady state condition $Db=De=D\pi=0$, which gives[12]

$$0 = H.\begin{bmatrix} de_s \\ db_s \\ d\pi_s \end{bmatrix} + A.dx \tag{7.15a}$$

where H is equal to the H-matrix in equation (7.11), x a vector representing all other determinants of the system and A the corresponding matrix of partial derivatives. Rewriting equation (7.15a) we find the steady state effects

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[12] In short-term analysis it is sometimes suggested that the so-called McKinnon-Oates condition is sufficient for equilibrium in the case of perfect capital mobility and $\sigma'=0$. This condition states that - for stationary equilibrium - the deficit in the government's budget should equal the deficit in the balance of payments on current account. Turnovsky (1976) has pointed out rightly that this condition is in fact a sufficient condition only in some very specific cases. More generally, conditions are required to hold for both variables separately. This is confirmed by the present analysis. This can be shown as follows. In our model the proper equivalent of the McKinnon-Oates condition appears to be $De+Db=0$. Since also $D\pi=0$ and all other variables are assumed to be constant, it can be seen from equation (7.11) that this condition is fulfilled only if both $De=0$ and $Db=0$ or if the H matrix satisfies

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = -H.\begin{bmatrix} -1 \\ 1 \\ d\pi/db \end{bmatrix}$$

On closer view this latter possibility should, however, be discarded for it is found to be satisfied only if $n-r=-1$, which is not only very unrealistic, but also in contradiction with the first order conditions for stability above.

$$\begin{bmatrix} de_s \\ db_s \\ d\pi_s \end{bmatrix} = -\mathbf{H}^{-1} . \mathbf{A} . dx \tag{7.15b}$$

In discussing this relation we shall follow the same procedure as before, first restricting the analysis to the simpler specific case with $\sigma'=0$ and thereafter dealing with the general case on the basis of some numerical results. In addition, the analysis will be confined to the impact of the instrument variables of the government g, τ_1, τ_2 and p.

Solving equation (7.15) for $\sigma'=0$ it is obtained that

$$\begin{bmatrix} de_s \\ db_s \\ d\pi_s \end{bmatrix} = \frac{1}{\mathrm{Det(H)}} \begin{bmatrix} \vartheta_\pi i_\pi(n-h_4) & -\vartheta_\pi i_\pi h_2 & h_2h_6-(n-h_4)h_5 \\ -\vartheta_\pi i_\pi h_3 & \vartheta_\pi i_\pi(n-h_1) & h_3h_5-(n-h_1)h_6 \\ 0 & 0 & (n-h_1)(n-h_4)-h_2h_3 \end{bmatrix} .$$

$$. \begin{bmatrix} -1 & c_1(y-\pi) & c_2y_2-(l+e)i_\tau & c_2(1-\tau_2) \\ 1 & -(y-\pi) & -y_2-(b+m)i_\tau & -(1-\tau_2)m \\ 0 & 0 & -\vartheta_\pi i_\tau & 0 \end{bmatrix} . \begin{bmatrix} dg \\ d\tau_1 \\ d\tau_2 \\ dp \end{bmatrix} \tag{7.16}$$

If the system is stable (hence Det(H) > 0 and n-r > 0) and $\tau_2 \geq \tau_1$ it can be assessed that the structure of these matrices is

$$\begin{bmatrix} + & - & ? \\ - & + & ? \\ 0 & 0 & + \end{bmatrix} . \begin{bmatrix} - & + & + & + \\ + & - & ? & - \\ 0 & 0 & + & 0 \end{bmatrix}$$

and for the product matrix[13]

$$\begin{bmatrix} de_s \\ db_s \\ d\pi_s \end{bmatrix} = \begin{bmatrix} - & + & + & + \\ + & - & - & - \\ 0 & 0 & + & 0 \end{bmatrix} . \begin{bmatrix} dg \\ d\tau_1 \\ d\tau_2 \end{bmatrix}$$

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[13] For $de_s/d\tau_2$ it is obtained that

$de_s/d\tau_2=(n-h_4)c_2y_2+h_2y_2-h_2(\tau_2-\tau_1)i_\tau/i_\pi-\{c_1(1-\tau_1)-c_2(1-\tau_2)\}(n-h_4)i_\tau/i_\pi$

Since $n-h_4,h_2,y_2>0$ and $\tau_2 \geq \tau_1$ this expression must always be >0. Similarly it is found for $db^*/d\tau_2$ that

$db^*/d\tau_2 = h_3c_2y_2-(n-h_1)y_2+\{c_1(1-\tau_1)-c_2(1-\tau_2)\}h_3i_\tau/i_\pi+(\tau_2-\tau_1)(n-h_1)i_\tau/i_\pi<0$

Thus both tax rates τ_2 and τ_1 and the 'inflation tax' p have a positive impact on e_s and a negative impact on b_s. Government expenditure has just the opposite effects. These results are quite straightforward and need little further comment. An interesting result is that none of these instruments except τ_2 affects the steady state profit rate π_s. This is due to the fact that the interest rate is given, so that π_s only depends on the natural growth rate, the given interest rate and the tax rate on profits.

A similar structure of the product matrix is found for the general case with $\sigma'<0$ for plausible values of the parameters. For n=6% and r=4%[14] it is obtained that (expressing e and b now in ratios to y)

$$\begin{bmatrix} de_s/y \\ db_s/y \\ d\pi_s \end{bmatrix} = \begin{bmatrix} -33.2 & 24.2 & 5.2 & 0.6 \\ 43.5 & -33.7 & -9.0 & -1.9 \\ 0.3 & -0.2 & 0.05 & -0.06 \end{bmatrix} \cdot \begin{bmatrix} dg/y \\ d\tau_1 \\ d\tau_2 \\ dp \end{bmatrix}$$

All coefficients have the same sign as in the $\sigma'=0$ case above. The principal difference concerns the last row, which now shows positive effects on the profit rate of government expenditure g and tax on property income τ_2 and negative effects of τ_1 and p. These effects arise from the impact of these variables on e and thereby, since $\sigma'\neq 0$, on the interest rate. Apart from this indirect influence τ_2 has a direct effect on π, for at a higher τ_2 the (gross) profit rate needs to be higher in order to maintain growth equilibrium.

It should be noted that the absolute size of the coefficients in the product-matrix is strongly influenced by the growth rate and the interest rate. For instance, if the interest rate is 2% instead of 4% a change in g by 0.01 would have led to an increase in public debt of only 0.06 instead of 0.44 as in the matrix above. And if the growth rate is 8% instead of 6% the change in public debt would reduce from 0.44 to 0.24.

7.8 A CURRENT BALANCE REGIME

In all the foregoing analysis it was assumed that the government adopts a certain target for its fiscal policy irrespective of the outcome for the balance of payments. In this respect we followed the tradition of most GBC literature. However, this assumption does not seem very realistic, as in most countries the position of the balance of payments on current account is an important guideline of fiscal policy. For example, until the end of the 1970's budgetary policy in the Netherlands was formally directed at a structural norm of a one percent surplus in the balance of payments on current account. Besides, it is well-known for many countries, that the urge for fiscal

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[14] All parameter values are the same as in figure 7.1a. Further it is assumed that π=0.10 and i_τ=-0.05.

restraint is, in practice, much stronger in the case of a deficit in the current account then when it shows a comfortable surplus.

In this final section we shall examine the (extreme) case in which fiscal policy is fully directed at maintaining equilibrium in the current account. Assuming that the initial net stocks of foreign assets is nil as well, this policy constraint implies that f=0 and De=0. If the government achieves this target by varying its expenditure g it can be seen from equation (7.5) that g must satisfy

$$g = y - c - i \tag{7.17}$$

Substituting this relation in the GBC the linearized differential system becomes

$$\begin{bmatrix} Db \\ D\pi \end{bmatrix} = \begin{bmatrix} h_1-n & -h_5 \\ 0 & \vartheta_\pi i_\pi \end{bmatrix} \cdot \begin{bmatrix} b-b_s \\ \pi-\pi_s \end{bmatrix} \tag{7.18}$$

where $h_1 = (1-c_2)(1-\tau_2)r-c_z$

$h_5 = (1+b+m)i_\pi+\{(1-c_1)(1-\tau_1)-(1-c_2)(1-\tau_2)\}$

The Routh-Hurwitz conditions of this system are

$$\begin{aligned} &\text{I. Trace (H)} = -(n-h_1) - \vartheta_\pi i_\pi < 0 \\ &\text{II. Det (H)} = (n-h_1).\vartheta_\pi i_\pi > 0 \end{aligned} \tag{7.19}$$

which implies after substitution for h_1 that

$$n-(1-c_2)(1-\tau_2)r + c_z > 0 \tag{7.20}$$

$$\vartheta_\pi i_\pi > 0$$

From these results it can be concluded immediately that under this balance of payments regime the system is more stable than under the original budgetary regime which required n>r (eq. 7.14).

The dynamics of this case is illustrated in the phase diagram below (*figure 7.3*) The {Db=0} curve is discontinuous at $\pi=\pi_1$ because at that profit rate the growth rate just equals the minimum growth rate n_{min}. The locus of the vertical {$D\pi$=0} curve is determined by the profit rate at which the corresponding growth rate is just equal to the natural growth rate n. As in that case the rate of unemployment is constant the income distribution will be constant as well. From the investment function (eq. 7.4) it can be inferred that the steady state profit rate π_s depends on n, r, τ_2 and ς. As regards the stability of the system it can easily be seen that the system is stable if $\pi_s>\pi_1$ and hence $n>n_{min}$ (*fig. a*) and unstable if $\pi_s<\pi_1$ (*fig. b*).

Figure 7.3 Phase diagram of b and π

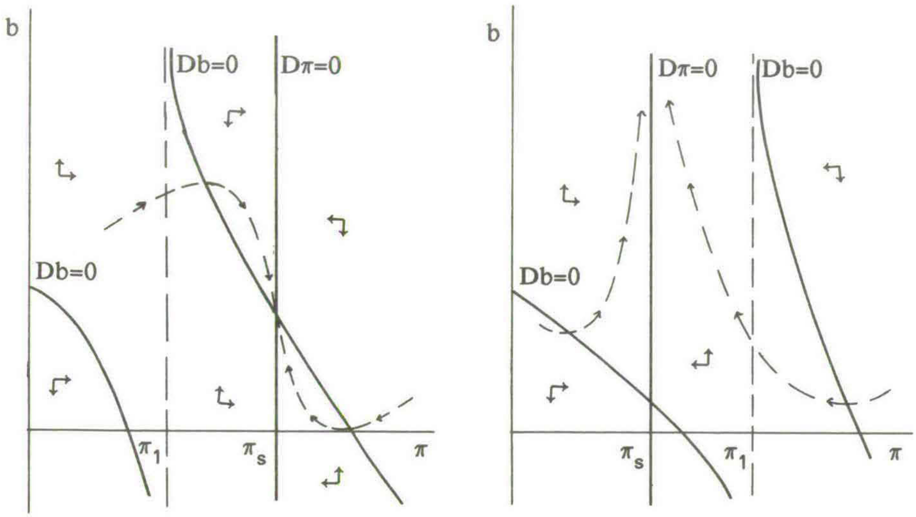
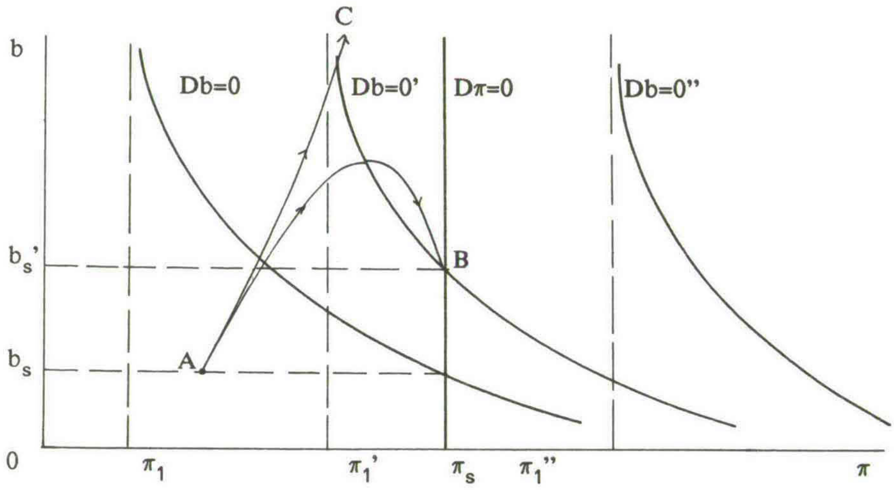


fig. a ($\pi_1 < \pi_s$) fig. b ($\pi_1 > \pi_s$)

Policy conclusions

On a very abstract level this model may illuminate some of the recent experiences with fiscal policy in the Netherlands and other European countries. As was described in the foregoing chapter, in the mid 1970's these countries were confronted by several (external) shocks that on the whole had a serious destabilizing effect on the growth of public debt. In addition, in the beginning of the 1980's the real rate of interest rose along with the international level of interest rates under the influence of a changed mix of monetary and fiscal policy in the United States. The overall outcome of these stocks may be discussed with reference to *figure 7.4*. Let us assume that the economy was originally in its stable steady state equilibrium (b_s,π_s). Then, as a result of the fall in ζ and the rise in r, the {Db=0} curve shifts to the right as higher profit rates are necessary to ensure Db=0. The effect of the changes in ζ, r and n has a negative impact on π_s, while the changes in ζ and r tend to raise π_s. For simplicity let us therefore assume that these different effects neutralize each other, so that the {Dπ=0} curve is unchanged. As finally the exogenous fall in π implies a movement away from the original equilibrium, the initial position of the system after the 'shocks' is characterized by point A right of the original equilibrium.

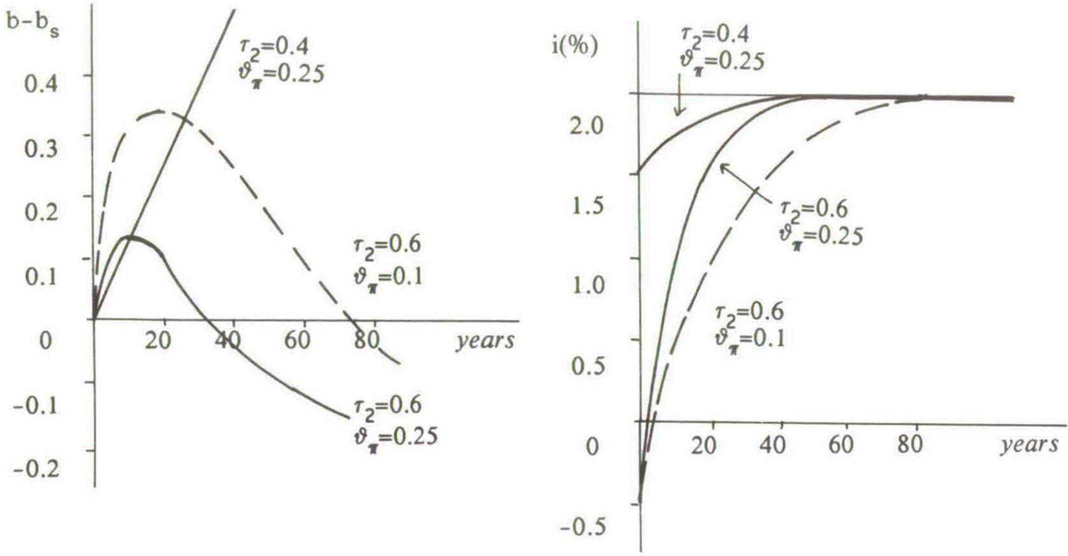
Figure 7.4 Shift of the Db=0 curve



Provided that the system is still stable ($\pi_1<\pi_s$) the adjustment trajectory is described by the A-B curve, which shows that after a period of rising debt the system will eventually tend to a new stable equilibrium where the profit rate was recovered back to its original level and public debt has reached a higher but stable level. The specific shape of the adjustment trajectory can be explained as follows. As a result of the shocks the growth rate and thus the volume of investment fall to a lower level. Then, in order to avoid the emergence of a surplus in the current account the government has to raise its expenditure discretely. Because of the lower growth rate and the higher expenditure public debt starts to increase sharply. Later, as the profit rate and the growth rate recover this rising tendency is mitigated and may even turn into steady decline.

Next imagine that the rise in r and the fall in n are such that the system becomes unstable. In *figure 7.4* this is shown by a further shift of the {Db=0} curve to the right. Then the adjustment process will follow the explosive A-C path. Although along this path the profit rate again tends to its stable level (π_s), public debt keeps growing till infinity. It may be noted that for government expenditure this trajectory entails the somewhat paradoxical result that after the initial discretionary increase in public expenditure, the government has to bring down its expenditure continuously. This follows from the balance of payments target which implies that as private

Figure 7.5 Adjustment trajectory of b and i



Explanation: this figure is based on a dynamic simulation of the linearized system (eq. 7.18). Before t=0 steady state equilibrium is characterized by n=2%, r=4%, τ_2=0.4 and all other variables as in fig.1a. At t=0, r rises to 5% thereby making the system unstable (n=2% < n_{min}=2.4). If τ_2 is raised to 0.6 stability is again restored (n=2% > n_{min}=1.6%).

investment and consumption rise (because of the growing interest income), public expenditure must be reduced equivalently. Since debt grows without any limit it can easily be seen that sooner or later public expenditure will become fully 'crowded out' by the rise in private expenditure, and thereafter should even have to become *negative*, which is of course impossible.

How is this unstable process to be stopped? There are several options. The first and by far the most attractive option would be to raise the natural growth rate to such a level that the system becomes stable again. However, it is obvious that this option may be hard to achieve in reality. A second option is to raise taxes on wages. This may temporarily mitigate the 'crowding out' of government expenditure by shifting the burden to wage earners, but since it does not cure the instability of the system this cannot provide a permanent solution. A third option is to raise taxes on private property income. This is a much more attractive option, for it not only relieves the burden of the government but also reduces the instability of the system. If the tax rate is raised above the critical level $\tau_2=1-(n+c_z)/\{1-c_2)r\}$ (equation 7.20) the process will even become stable again.

Although this last option is most effective from a *long term* point of view, it is doubtful whether it will also be a politically attractive option because it leads to a *lower* growth rate of production and thus maybe even to a steeper rise in public debt in the *short term*.[15] This is illustrated in *figure 7.5* which shows the adjustment trajectory after an increase in r for the case in which τ_2 is left unchanged (τ_2=0.4), and for the case where τ_2 is raised to 0.6 in order to restore stability. As this figure brings out the second (stable) alternative implies lower growth and higher debt in the 'short' run. The duration of this short run depends mainly on the difference between the growth rate and the interest rate and on the speed of adjustment of the profit rate (ϑ_π). If ϑ_π is low this 'short' run may be quite long (as is shown by the figure). But even for higher ϑ_π's the short run seems to last decades rather than years; this may be too long for governments with a really short time horizon (for example because of periodic elections).

7.9 CONCLUSION

In this chapter we have analysed the dynamics of growth and debt for a small open economy. The conclusions of the present analysis are most clear-cut for the case of a purely exogenous interest rate (σ'=0). Then stability requires for the reference regime (Blinder-Solow) simply that n-r>0. This result implies that neither fiscal nor monetary policy can influence the long-term stability or instability of the system. This rather pessimistic result is modified in the more general case with an endogenous interest rate (σ'>0), but even then fiscal and monetary policy instruments prove to have only a very small impact or no impact at all on the system's stability. The growth rate and the interest rate emerge again as the fundamental determinants of stability.

In general for other policy regimes less stringent conditions for stability were found. A necessary condition common to all alternative regimes was that n must satisfy $n-(1-c_2)(1-\tau_2)r+c_z>0$, which implies that the government has at least one important instrument by means of which it might effectively influence the stability

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[15] The impact effect of τ_2 on b is obtained by differentiation of the GBC:

$$dDb/d\tau_2 = (1+b+m)i_\tau - (1-c_2)y_2.$$

If the sensitivity of i for τ_2 is substantial this impact might well be positive. The long term effect of τ_2 on b* is found to be

$$dDb^*/d\tau_2 = -(n-h_1)^{-1}\{(1-c_2)y_2 - ((1-c_2)(1-\tau_2) - (1-c_1))i_\tau/i_\pi\}$$

Since i_τ is probably much smaller then i_π in absolute terms, this long term effect will generally be negative.

of the system, namely the tax rate on property income τ_2.

All policy regimes which are commonly considered in the GBC literature focus on norms for government outlays, or taxes, irrespective of their consequences for the balance of payments. In this chapter we have argued that this is inappropriate, because in practice the balance of payments on current account is an important guideline for fiscal and monetary policies. Therefore we have examined a regime with a zero current balance target. The conditions for stability were found to be $\vartheta_\pi i_\pi>0$ and $n-(1-c_2)(1-\tau_2)r+c_z>0$, which implies that in this regime the stability can be influenced effectively by the government through manipulation of the tax rate on interest income τ_2. Monetary policy, aiming at a certain target rate of inflation or depreciation, is once again ineffective in this case.

APPENDIX 7.4 ALTERNATIVE REGIMES

This appendix establishes the stability conditions for the alternative policy regimes given in section 7.4. In all other regimes it is assumed that the target of fiscal policy is achieved by varying g. This yields the following conditions for g:

a. Tobin-Buiter $g= g_o-(1-\tau_2)(r+p)b$
b. Christ $g= g_o-(r+p)b$
c. Domar $g= g_o+\tau_1 w+\tau_2 y_2 - (r+p)b$
d. Buchanan $g= \tau_1 w + \tau_2 y_2-(r+p)b$
e. Barro $g= \tau_1 w+\tau_2 y_2 + (i+p)(b+m) - (r+p)b$

These functions have the following consequences for the elements of the first two rows of the H matrix.

new coefficient	Buchanan	Domar	Tobin-Buiter	Christ
$h_1' =$	$h_1-\tau_2 r + \sigma'(b-\tau_2 b-\tau_2 e)$	$h_1-\tau_2 r+\sigma'(b-\tau_2 b-\tau_2 e)$	h_1	h_1
$h_2' =$	-	$h_2+\tau_2 r+(r+p)$	$h_2-(1-\tau_2)(r+p)$	$h_2-(r+p)$
$h_3' =$	-	0	h_3	h_3
$h_4' =$	-	$-p$	$h_4-(1-\tau_2)(r+p)$	$h_4-(r+p)$
$h_5' =$	$h_5+(\tau_2-\tau_1)$	$h_5+(\tau_2-\tau_1)$	h_5	h_5
$h_6' =$	-	$h_6-(\tau_2-\tau_1)$	h_6	h_6

The stability conditions for the regimes a.- c. are obtained by substituting the above new coefficients in the Routh-Hurwitz conditions, which yields for the Domar regime and for the Christ regime

(I) $n-p > 0$

(II) $n-(1-c_2)(1-\tau_2)r+c_z > 0$

and for the Tobin-Buiter regime

(I) $n-\tau_2 r+(1-\tau_2)p > 0$

(II) $n-(1-c_2)(1-\tau_2)r+c_z > 0$

In the balanced budget regime (d.) the growth of public debt is given by

$$Db = -(n+p)(b+m)$$

which implies that in the steady state (b+m) must be zero and thus b=-m. This reduces the model to two independent equations. For the specific case with $\sigma'=0$ this regime is found to require for stability

$$n-h_1-\tau_2 r > 0$$

or after substitution for h_1

$$n-(1-c_2)(1-\tau_2)r+c_z > 0$$

besides, of course, the stability condition for income distribution $\vartheta_\pi i_\pi>0$.

For the Barro regime where b=constant the system reduces to two dimensions. The new elements of the H matrix are now

$$h_1' = h_1 - \tau_2 r + \sigma'\{\tau_2(b+e) + (b+m)i_r - b\}$$

$$h_5' = h_5 + \tau_2-\tau_1 + (b+m)i_\pi$$

while h_2, h_3, h_4, h_5 are now equal to zero. For the reduced model with $\sigma'=0$ this implies the condition $h_1'-n>0$, thus again

$$n-(1-c_2)(1-\tau_2)r+c_z>0.$$

These results are presented in *Table 7.1* in the text.

CHAPTER 8

CONCLUSION

This book investigates the dynamics of growth and debt on the basis of a post-Keynesian model of growth and income distribution. Particular attention is given to the dynamics implied by the government budget constraint.

The dynamics of government finance is usually analysed on the basis of a neoclassical-Keynesian IS/LM model. This model which concentrates on income-expenditure equilibrium neglects the distribution of income, and is essentially suited for short-period analysis. The evolution of public debt is, however, typically a long-period phenomenon. Therefore it is more appropriate to analyse the GBC dynamics in the context of a growth model. Moreover, this model should also take account of the accumulation of debt and wealth in the private sector. In this book the dynamics of public debt is examined in relation to the distribution of wealth and debt between two distinct classes in the private sector. This last chapter also investigates the relationship of the dynamics of public debt with the growth of foreign debt and wealth.

8.1 POST-KEYNESIAN THEORY

Starting from a simple generalized Pasinetti-Kaldor model with two social classes (workers and capitalists), chapter 2 shows that the introduction of the government budget constraint raises serious difficulties with regard to the long-period solution of this model. In the presence of a government sector the steady state solution proved either to be unstable or to be characterized by a net *creditor* position of the government and the disappearance of the capitalist class. Apparently the traditional (linear) post-Keynesian model is too rigid to provide a reasonable description of the long-term dynamics of the economy. Therefore it was necessary to develop a more sophisticated model.

Differential saving

Despite the fact that the proposition of differential saving is, according to Malinvaud (1986), accepted by most macroeconomists, it still lacks a rigorous theoretical explanation. There seem to be two opposing views on this subject among post-Keynesian economists. According to Kaldor the higher savings propensity from profits arises from the 'nature of business income', while Pasinetti explains it by the stronger tendency of the rich to plan for inheritance. Thus in Kaldor's view differential propensities to save are attached to the different types of income, while Pasinetti, following the classical approach in this respect, relates them to the different preferences of social classes.

Kaldor's view is supported by Malinvaud (1986), who emphasizes that changes in business income have a smaller impact on consumption than equivalent changes in wage income because of informational imperfections and liquidity constraints. These are, however, essentially short-period factors. In our analysis we follow the 'classical' view of Pasinetti that differential saving is a class related phenomenon.[1] Nevertheless we agree with Kaldor that differential saving has something to do with the organization of business. However, this is not because people cannot see through a 'corporate veil', but because there exists a 'class' of owners and (top-)managers of the corporate sector with a distinct role in the economy and who therefore have different interests with respect to saving and wealth. While for workers saving is primarily a provision for their old age, for members of the 'corporate class' it is essentially a means of increasing their wealth, and thereby their status and power.

Our analysis therefore adopts a savings function which distinguishes between two classes, workers and a 'corporate class' of large stockholders and managers of firms. Workers are assumed to be risk averse and to save for their old age and inheritance only. For simplicity, it is assumed that workers do not own shares and invest all their wealth in interest-bearing debt. All shares are owned by the corporate class which bears the risks of enterprise and appropriates the profits. For this class, which as a whole owns and governs the capital stock and the productive capacity, saving and wealth is an aim in itself.

Of course, this division of the economy into two classes is schematic. A further improvement would be to disaggregate this corporate class into several sub-classes. This would, however, have significantly complicated our analysis. As our focus was on the macroeconomic dynamics rather than on the precise relationship between the ownership and the control of firms, this elaboration was beyond the scope of our analysis. On the (microeconomic) level of the firm it is shown that the strategy towards growth and finance can be explained by the conflicting interests of managers and shareholders, and the discretionary power of the management vis-à-vis the shareholders.

Thus our analysis distinguishes four sectors, or agents:

1. *workers* who receive labour income, transfer-income from the government and interest-income on their accumulated savings. This class is risk averse, does not own shares, and saves for old-age and inheritance only;

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[1] In chapter 7, which adds the foreign sector to the model, we have for practical reasons adopted a Kaldorian savings function with differential propensities to save related to types of income rather than to classes. This was necessary in order to reduce the dimensionality of the model.

2. the *corporate class*, which encompasses all entrepreneurs, shareholders and (top)managers of firms. This class is risk-taking, it owns all shares and appropriates the profits after payment of interest on debt to the workers and taxes to the government.
3. the *government sector* which receives taxes levied on labour income, interest income and net returns of the corporate sector. It finances its deficit by issuing money and interest-bearing debt.
4. the *foreign sector* which pays interest on its debt to domestic sectors, or receives interest if the country is a net debtor. The deficit or surplus in the balance of payments on current account is financed fully by interest-bearing debt. There is no net flow of official reserves. The monetary authorities are assumed to adjust the money supply to the demand for money at the given rate of inflation, and depreciation.

Managerial growth

The determinants of growth and investment are investigated on the basis of a model of an individual, equity-rationed, firm. Chapters 3 and 4 show that in the presence of financial constraints, arising from imperfect markets for risk-sharing, investment can be explained in terms of internal savings and the desired rate of indebtedness. Thus investment depends on profits in two ways: as a source of finance (through the flow of internal savings) and as an incentive for taking risks (through the desired debt ratio).[2]

In many respects our analysis follows the managerial or corporate approach to growth. An interesting feature of this approach is that it distinguishes between the interests of managers and the interests of shareholders of the firm. It is generally assumed that managers are mainly interested in the expansion of the firm[3] in contrast to shareholders who desire a maximum market value of their shares. The actual strategy of the firm depends therefore on the discretionary power of the management vis-à-vis the shareholders. We have considered two ways to model this conflict of interests, one focussing on the shareholders and the other on the managers. For shareholders the discrepancy between the managerial strategy and the strategy which maximizes the valuation of shares provides an incentive for monitoring. As the cost of monitoring must be subtracted from the net pay-out of profits, the optimum monitoring effort is found where the marginal benefits in terms of a more 'shareholders minded' strategy are offset by the marginal cost of monitoring (section 3.6).

[2] In this respect our model is broader than Malinvaud (1980), who concentrates on the incentive role of profits: "The main concern has been profitability as a precondition for risk taking by entrepreneurs; the model cannot do much more than explore the consequences of such a precondition. Others will have to study the role of financial constraints," Malinvaud (1980, p. 101).

[3] Or in perquisites, Jensen and Meckling (1976).

The second approach starts from the point of view of managers and emphasizes the role of the market-valuation of shares as an indicator of the risk of intervention by shareholders or take-over by others. Following Odagiri (1981) it is assumed that this risk increases as the discrepancy between the actual market-valuation and the maximum valuation becomes greater. Then, by discounting this risk in the managerial optimization procedure it is possible to establish a unique optimum growth strategy for the firm depending on the preferences of managers and shareholders, the chance of intervention by shareholders or take-overs, and the mean and the variance of profits, taxes and the interest rate.

8.2 THE METHOD OF THE MACROECONOMIC ANALYSIS

For the analysis of the macroeconomic dynamics we have adopted a sequential-analytical approach. According to this method the analysis is divided into different levels corresponding to the different lengths of the periods (cf. Malinvaud 1980, Kuipers 1981). Our analysis concentrates on the medium period and the long period. The medium period is conceived as a (continuous) sequence of short-period ('momentary') equilibria, and the long period as a sequence of medium-period equilibria.

The models constructed are in general simple non-linear models. Our main challenge has been to construct models that are consistent not only in the neighbourhood of equilibrium but also for positions farther away from equilibrium. This is necessary to determine the global dynamics of the system. This is relevant unless one believes that the economy is always in or close to its ('natural') equilibrium. An additional complication of non-linear systems concerns the uniqueness of equilibrium. Our analysis shows that even simple and natural non-linear systems may yield more than one solution. As a consequence the actual evolution of the economy becomes critically dependent on its initial position. Moreover, it can no longer be assumed that after a shock the system will always return to its original equilibrium. It might well tend to another equilibrium, or develop into an unstable process which does not lead to a new equilibrium at all.

Because of the complex nature of non-linear dynamic systems, it is necessary to keep the models simple. The dynamics of two-dimensional systems of differential equations (quadratic functions) is reasonably well-established, but even three-dimensional (cubic) systems soon become hard to deal with mathematically. As our main concern was the theory and not the technique of modelling non-linear systems, we have constructed our models in such a way that, although they often could not be solved explicitly, they remained transparent from an economic point of view.

8.3 THE DYNAMICS OF THE GOVERNMENT BUDGET CONSTRAINT

Short period

Although we have not explicitly modelled short-period dynamics, it is shown that the distribution of wealth and debt may also have important consequences for short-period equilibrium. It is well-known that a change in the price-level changes the distribution of wealth between debtors and creditors, and will therefore have an impact on aggregate demand. Tobin (1980) pointed out that this may give rise to a *reverse Pigou effect*: that is, a general price rise causes expenditure to *rise* rather than to fall. Similarly our analysis suggests that also a change in the interest rate may have a reverse impact on expenditure and savings. This is due to the redistribution of income from the debtor sectors (government and the corporate sector) to the creditor sector (workers). Especially in the presence of a large public debt this may lead to a *reverse Cambridge effect*: that is, a higher interest rate leads to a *decrease* of aggregate savings. If this distribution-effect is stronger than the conventional effect of the interest rate on investment, a higher interest rate may even yield a fall in aggregate expenditure (this may be called a *reverse Keynes effect*).

It is evident that these distribution effects may seriously affect the equilibrium restoring role of the price level and the interest rate, and will therefore have a strong impact on the stability of short-period and medium-period dynamics.

Medium period

The medium period is characterized by sluggish prices and disequilibrium between aggregate demand and supply. In this context the decisive determinant of investment is the aim to adjust capacity to demand. This gives rise to a Harrod type of dynamics. Our analysis in chapter 5 shows that the 'knife-edge' instability of Harrod's model is mitigated significantly if one takes account of a certain degree of price-flexibility and feedback from a non-accommodating monetary sector. Nevertheless, even if this feedback leads to a stable equilibrium, it turns out to be stable only within a certain zone around this equilibrium, that is, it is locally stable but not globally. Numerical experiments indicate that this zone may take the form of an *unstable limit cycle*. Starting from a point inside this cycle the system will return to its equilibrium, but if it starts outside the cycle the system will recede from it for ever.

In economic terms this gives support to Leijonhufvud's proposition that Keynesian dynamics is characterized by a '*corridor*': within this corridor the system is self-stabilizing, but beyond it the disequilibrium, or 'deviation-amplifying', forces become so strong, that the system will never return to equilibrium by itself.

Whether a locally stable solution, and thereby a corridor, exists depends primarily on the response of investment to discrepancies between demand and capacity. If this acceleration factor is too strong the solution becomes unstable, locally as well as globally. There thus exists a critical value for the adjustment speed of investment beyond which the solution becomes unstable. In technical terms this critical parameter value represents the *bifurcation point* of the catastrophe manifold of the system (section 6.3). We have used the critical value of this parameter in order to asses the impact of different variables on the stability of the system.

Long cycles

The long-period analysis concentrates on the dynamics of growth, income distribution and the accumulation of debt and wealth. As the medium-period results were not very encouraging as regards the intrinsic stability of the system, it is assumed that in the long period equilibrium between aggregate demand and supply is continuously being ensured by monetary policy. In the long period monetary authorities are thus assumed to have enough time to find the right policy to achieve full capacity utilization. The long-period dynamics is governed therefore by the interaction between labour market disequilibrium and the distribution of income, on the one hand, and the accumulation of assets ensuing from the budget constraints of the government, the workers and the corporate sector, on the other hand.

If the interaction between labour market disequilibrium, wage growth and investment is the dominant mechanism, it is shown that this may lead to a 'Goodwin' like cycle. More interesting in the context of our analysis is that financial factors may also cause a long swing in economic activity. In the presence of financial constraints, in particular constraints with respect to equity finance, this cycle is governed by the interaction between investment, internal savings and the evolution of the (desired) rate of indebtedness. Because investment now depends on the flow of internal savings as well as the discrepancy between the actual and the desired debt ratio, a shock to the system, for example a fall in profits, will have a double impact on investment: first, through the change in internal savings, and secondly, through the change in the desired debt ratio. The analysis of the adjustment process for an individual firm (chapter 5) reveals that this second factor especially may give rise to a lasting process of financial adjustment characterized by initial 'overshooting' of investment.

On an abstract level this may clarify some aspects of the developments after the oil-shock in the beginning of the 1970's. This shock not only affected the profitability of investment, but also the general state of uncertainty and thereby the desire for growth (the 'animal spirits') of entrepreneurs. After the (over-) optimistic 1960's there has been an adverse shift in business confidence leading to more prudential financial policies. As a result investment fell not only because of the fall in internal savings, but because of the desire to reduce the rate of indebtedness as well. Notably this structural shift in the state of confidence may explain why recovery after the mid-1970's was so hesitant and why the recovery of investment lagged so much behind the improvement of actual profitability. These phenomena could not be explained by real factors alone. One should take account of financial constraints as well.

Open economy

In an open economy the domestic dynamics of wealth and debt should be considered together with the evolution of the external debt and wealth of domestic agents. In addition to the cumulating interest payments on public debt, now net interest payments on the external position may also produce a tendency towards instability. Through interest payments creditor countries tend to become even stronger creditors, and debtors ever larger debtors. Chapter 7 shows that for a small open economy the growth rate and the interest rate are the primary determinants of (local) stability. If domestic and foreign assets are perfect substitutes, the real interest rate in a small open economy is fully determined by the international interest rate. Hence, the stability of public debt depends crucially on the domestic rate of growth. Slowly growing economies will therefore be more liable to financial instability than similar economies with a high rate of growth. If, as a result of imperfect substitution, the domestic interest rate is dependent on the size of the net external position, this tends to increase the intrinsic instability for debtor countries. For creditor countries the risk of instability tends to be less[4].

The analysis of the open economy has been restricted to the small open economy. It neglects the interaction between growth and asset accumulation in different countries. In this respect our analysis is still deficient. It would be interesting to extend the present analysis to a two-country model, or even to generalize it for *more country models*. This would require a synthesis of the analysis for the closed economy (chapters 5 and 6) and the analysis of the open economy in chapter 7. One interesting feature of an integrated world model is that changes in prices, exchange rates and interest rates would cause distribution-effects on a world scale as well. As a rise in the interest rate benefits creditor countries vis-à-vis debtor countries, it depends on the marginal saving propensities in different countries whether this leads to a rise or fall in aggregate expenditure.

Obviously, these models will be very complex and hard to deal with on the analytical level. Nevertheless, it would be very interesting to investigate how these models would behave in the long term. It is clear that a global steady state solution can exist only under very restrictive conditions. As steady growth requires all stocks and flows to grow at the same rate in real terms it is obvious that the net external position of any country must be zero in equilibrium, unless countries happen to grow at the same rate. Both conditions are very specific, and no obvious process seems to exist which would realize either of these conditions.[5] Hence, it appears that a steady state

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[4] See also Van Ewijk (1983).

[5] There is no natural tendency towards a zero net external position. This can be shown as follows. In comparison with our model for a small open economy, the two-country case introduces one extra constraint, namely equilibrium between demand and supply on a world scale, and one extra free variable, the interest rate. Therefore the model is fully determined for any initial external position. Next assume that one steady state condition is satisfied, namely the net external position being zero. Then it is evident that this external position can remain zero over time only if the balance of payments in current account is zero as well. That

equilibrium can be realized only in the presence of international coordination of fiscal policies, i.e. if the fiscal policy regimes are consistent with a zero current account and a zero net debtor, or creditor, position in the long term.

Determinants of stability

There is an essential difference between the determinants of stability in the medium period and the long period. In the medium period, which is characterized by the disequilibrium dynamics between demand and capacity, all factors that tend to stabilize demand also have a stabilizing impact on the system as a whole. Thus all factors that raise the autonomous part of spending are intrinsically stabilizing. These factors include the autonomous inflation and money growth (which determines the equilibrium rate of inflation). Further, all factors that strengthen the monetary feedback are stabilizing, in particular the degree of price-elasticity and the sensitivity of the interest rate with respect to real money stock.

In the long period the natural rate of growth and the equilibrium rate of interest emerge as basic determinants of (local) stability. Therefore all factors that depress the equilibrium interest rate tend to have a stabilizing impact on the system. Thus in contrast with the medium-period dynamics all factors that *reduce* expenditure and increase saving are stabilizing.

Wealth elasticity of consumption

A general conclusion of many IS/LM based studies on the stability of the government budget deficit is that a high wealth elasticity is essential for stability. However, our analysis, which concentrates on a long-period post-Keynesian world, tends to support the opposite conclusion, namely that a high wealth elasticity of consumption has a destabilizing impact on the system. This can be explained as follows. In short-period IS/LM models, characterized by under-utilization, a higher wealth elasticity enhances the feedback of public debt on consumption, and thereby on production and income, and thus finally on tax receipts. Therefore a rise in public debt may lead to a *reduction* in the budget deficit if the wealth effect is sufficiently strong. In the long period when capacity is fully utilized, real tax receipts cannot be increased further through a rise in income. In that case, a stronger wealth effect only increases the impact of debt on the interest rate. Therefore a high wealth elasticity is generally destabilizing in our analysis.

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is, if in both countries domestic demand just happens to equal domestic supply at the given international equilibrium rate of interest. This would, however, be purely accidental, and must be excluded on logical grounds. Therefore, steady state equilibrium is possible only if the real rate of growth is the same for both countries, or if fiscal policy in both countries deliberately aims at a zero external position or a zero deficit on current account.

8.4 POLICY CONCLUSIONS

One of the central questions of our analysis concerns the impact of government policy, in particular fiscal policy, on the dynamic stability of the system. Fiscal policy is represented by a linear policy function. By varying the parameters this function encompasses all of the following regimes to be found in the literature:

1. *Blinder-Solow* regime, with fixed tax rates and fixed expenditure;
2. *Tobin-Buiter* regime, with fixed taxes rates and a fixed sum of expenditure and interest payments net of taxes;
3. *Christ* regime, with fixed tax rates and a fixed sum of expenditure and interest payments;
4. *Domar* regime, with a fixed target for the budget deficit;[6]
5. *Barro* regime, with a fixed target for debt as a ratio of national product.

These regimes have been investigated with regard to their impact on the stability of the system in the medium period, the long period and for the case of the small-open economy. In the medium term the Christ regime turns out to be the most stable regime, closely followed by the Tobin-Buiter regime. Both regimes are clearly counter-cyclical thanks to the fact that they imply a inverse relationship between expenditure and interest payments. As the interest rate is low in a depression and high during prosperity this produces a stabilizing variation in expenditure. The Barro regime, which aims at a constant debt ratio, performs by far the worst in the medium period. This is not surprising as this regime links government expenditure to two pro-cyclical factors: tax revenue and the 'real erosion' of debt. through the growth of production. As a result a rise or fall in economic activity will be magnified by the consequential rise or fall in government expenditure.

For the long period the results are different. Now the Barro regime proves to be the most stable regime; that is, with the smallest risk of a spiral of cumulating debt and interest payments. The second-best regime is the Christ regime which is obviously due to the strong feedback of debt service to expenditure. In the long period the Blinder-Solow, which assumes fixed expenditure as well as tax rates, turns out to be the most de-stabilizing regime.

The results for the open economy are basically similar to those for the long period. The precise ranking of the regimes appears to depend on the net external position, but in general the Blinder-Solow regime performs worst again and the Barro regime best. These results reveal a sharp distinction between the stability of the budgetary regimes in the medium period and in the long period. This is especially true for the Barro regime which is the most stable regime in the long period while the most unstable regime in the medium period. Therefore it is impossible to establish a unique optimum regime. Nevertheless it is evident that a regime which links the room for government

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[6] The Buchanan regime with a zero deficit can be considered as a special case of this Domar regime.

expenditure to the amount of debt service, has a stabilizing impact on the economy in the medium period as well as in the long period. Therefore, of the regimes considered in our analysis the Christ regime, which implies a one for one linkage of expenditure to interest payments, appears to offer a reasonable 'second best' solution from the medium-term as well as the long-term point of view.

In an open economy another factor must be taken into account, namely the evolution of the external position. As the change in the external position depends on the excess of private savings over domestic investment including the government budget deficit it may be sensible therefore to direct fiscal policy to a target for the balance of payments. In fact, the budgetary regime based on a structural deficit norm employed in the Netherlands from 1961 to the mid 1970's, was based implicitly on a norm for the balance of payments in current account (Sterks, 1984).

Our analysis has concentrated on the medium-term and long-term consequences of fiscal policy. No particular attention has been given to policies to cure disequilibrium in the labour market. As we have seen dynamic equilibrium does not automatically imply full employment. On the contrary, the rate of unemployment is such that it is compatible with a constant income distribution. Thus, just as with income distribution, unemployment depends on the dynamic equilibrium, and therefore on all factors that are relevant to the growth of the economy.

Although it is attractive, both on political and economic grounds, to adopt certain rules or norms for fiscal policy, our analysis shows that rules are valid only in a given structural environment. Analysis of our non-linear models revealed that the dynamics of the economy may change significantly as the system moves farther away from equilibrium. In the event of a shock the system might thus be displaced from the stable area into an 'unstable' area where it becomes subject to deviation-amplifying processes. Moreover, the dynamics of the system may also change as a result of a (gradual) evolution of the environment. Formerly stable policy regimes may therefore turn into unstable regimes in the event of a shock, but also due to a gradual change of the environment. Rules have therefore to be reconsidered regularly, and discretionary decisions may be necessary to change them. Thus not only does a change of the rules lead to a change of the system (Lucas, 1976), but also a change of the system must lead to a change of the rules.

LIST OF SYMBOLS

(All stock and flow variables are in real terms, and expressed as a ratio to capital stock, unless stated otherwise).

a	=	corporate debt
b	=	public debt
c	=	propensity to consume
C	=	total consumption
d	=	ratio between debt and net worth
e	=	net external creditor (+) or debtor position (-)
E	=	exchange rate
f	=	balance of trade
g	=	government expenditure
G	=	stock of goodwill
h	=	utilization rate
i	=	net investment
j	=	price change of shares
J	=	number of shares
K	=	capital stock (absolute, volume)
m	=	base money
M	=	base money (absolute, nominal)
l	=	employment (in efficiency units)
l_c	=	employment at capacity level
l_s	=	labour supply (in efficiency units)
n	=	growth of labour supply (in efficiency units)
p	=	inflation
P	=	price level
q	=	market valuation ratio
r	=	real interest rate
R	=	nominal interest rate
s	=	propensity to save
S	=	total savings
t	=	time
T	=	total taxes
u	=	rate of unemployment
U	=	unemployment (absolute)
v	=	managerial valuation ratio
V	=	net worth
w	=	wage income
W	=	real wage rate

y = net production
y_c = productive capacity
Y = production (absolute, volume)
z = wealth
Z = total market demand

α = risk aversion coefficient
β = labour elasticity of production
γ_i = fiscal reaction coefficient (i=1,..4)
δ = pay-out of profits
ϵ = rate of external financing
ζ = 'animal spirits' parameter
η = time preference
ϑ_x = adjustment coefficient ($x=i,g,\pi,..$)
θ = pay-out ratio
Θ = exchange rate
κ = cost of monitoring
μ_i = money demand parameters (i=1,2,3)
π = profit rate
ρ = discount rate
σ = risk premium
τ_i = tax rate (i=1,2,3)
ψ = depreciation rate

Dx = dx/dt
x_e = expected value of x
$\hat{x}$ = growth of x (in absolute terms)

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AUTHOR INDEX

OVER DE DYNAMICA VAN GROEI EN SCHULD

SAMENVATTING

De stagnatie van de economsiche groei sinds het midden van de jaren '70 heeft in veel westerse landen geleid tot aanzienlijke tekorten op de overheidsbegroting. Als gevolg daarvan is de staatsschuld sterk toegenomen; niet alleen in absolute bedragen, maar maar ook in verhouding tot het nationaal inkomen. Door de rente, die over deze schuld moet worden betaald, wordt de overheidsbegroting opnieuw belast. Hierdoor dreigt er een spiraal te ontstaan van groeiende rentelasten, toenemende tekorten en daardoor weer verder stijgende staatsschuld. In de jaren '80 is deze tendens versterkt door de, zeker voor historische begrippen, hoge reële rente.

Deze ontwikkelingen hebben in de macro-economische theorie geleid tot een verschuiving in de aandacht van de conjunctuurpolitiek op korte termijn naar de gevolgen van de overheidsfinanciën op de lange termijn. Dit heeft aanleiding gegeven tot een heftig debat over de effectiviteit van begrotingspolitiek indien men rekening houdt met de financiële gevolgen van het beleid op langere termijn.

Deze studie sluit aan bij dit debat, maar doet dit vanuit een andere invalshoek. In de eerste plaats staat niet de *effectiviteit* van de begrotingspolitiek centraal, maar de gevolgen ervan voor de *stabiliteit* van de economie. In de tweede plaats wordt gekozen voor een andere theoretische basis dan de gebruikelijke, namelijk het post-Keynesiaanse model van groei en inkomensverdeling, in plaats van het neoklassiek-Keynesiaanse model dat zich met name richt op de conjuncturele ontwikkeling op korte termijn.

Kenmerkend voor de post-Keynesiaanse benadering is de dynamische opvatting van de economie, en de centrale plaats die daarbij wordt gegeven aan de verdeling van het inkomen en het vermogen tussen verschillende sectoren, of sociale klassen, binnen de economie. Deze theorie verschaft een goede basis voor de analyse van de ontwikkeling van de staatsschuld in relatie tot de vermogensontwikkeling binnen de particuliere sector en de internationale schuldverhoudingen.

Deze studie is theoretisch van aard. Getracht wordt om op basis van een theoretische analyse inzicht te verschaffen in de dynamische verbanden tussen de begrotingspolitiek van de overheid en het spaar- en investeringsgedrag van de particuliere sector. Met name wordt onderzocht onder welke voorwaarden de economie stabiel is, d.w.z. tendeert naar een pad van gelijkmatige groei. Vanwege het complexe karakter van (niet-lineaire) dynamische systemen is het noodzakelijk om kleine en overzichtelijke modellen te kiezen. De modellen verschaffen derhalve slechts een gestyleerd beeld van de economie.

Na de inleiding (hoofdstuk 1) worden in hoofdstuk 2 de uitgangspunten van het

post-Keynesiaanse model behandeld. De nadruk ligt daarbij op de verklaring van de differentiële spaarfunctie. Volgens deze spaarfunctie hangt de omvang van de besparingen in de economie af van de verdeling van het nationaal inkomen over winstinkomen en looninkomen. In dit hoofdstuk worden de consequenties onderzocht voor het post-Keynesiaanse model wanneer het wordt uitgebreid met een overheidssector. Het blijkt dan dat het post-Keynesiaanse model in zijn 'klassieke' vorm te beperkt is om een goede en consistente beschrijving te kunnen geven van de ontwikkeling van een economie op lange termijn.

Omdat de ontwikkeling van de overheidsfinanciën nauw samenhangt de ontwikkeling in de particuliere sector, wordt in de hoofdstukken 3 en 4 een uitwerking gegeven van het investerings- en financieringsgedrag van bedrijven. Uitgangspunt voor deze analyse is dat er geen perfecte markt bestaat voor risicodragend kapitaal (aandelen). Als gevolg hiervan zijn bedrijven 'gerantsoeneerd' in hun eigen vermogen. Voor de financiering van hun investeringen zijn zij derhalve aangewezen op de eigen besparingen (ingehouden winsten) en op leningen door derden. In hoofdstuk 3 wordt aangetoond dat er een afweging bestaat tussen het tempo waarmee ondernemingen kunnen uitbreiden en het financiële risico dat zij daarbij lopen. Hoe deze afweging tot stand komt hangt mede af van de machtsverhouding tussen de managers, die de onderneming leiden, en de aandeelhouders, die formeel de onderneming bezitten. Dit wordt nader uitgewerkt in hoofdstuk 4, waarin tevens aandacht wordt besteed aan het verband tussen het groeitempo en de winstgevendheid van ondernemingen. Tenslotte geeft dit hoofdstuk een beschrijving van het (financiële) aanpassingsproces dat volgt na een verstoring van het evenwicht, bijvoorbeeld door een verandering van de winstmogelijkheden, of de rente.

Deze (micro-economische) theorie van de ondernemingstrategie dient als basis voor de analyse van de macro-economische dynamica in de hoofdstukken 5 - 7. De analyse wordt gesplitst in drie delen. Hoofdstuk 5 behandelt de dynamica van de economie op de middellange termijn, hoofdstuk 6 de dynamica op de lange termijn, en hoofdstuk 7 de dynamica van een economie in relatie tot het buitenland. Op de *middellange termijn* staat de wisselwerking tussen groei, investeringen en de bestedingsontwikkeling centraal. Uit deze analyse blijkt dat de economie gekenmerkt kan zijn door 'corridor' stabiliteit. Dit houdt in dat de economie stabiel is bij kleine schokken, maar dat de economie bij grotere schokken buiten zijn stabiele 'corridor' kan worden gebracht. In dat geval krijgen de-stabiliserende krachten de overhand, en kan de economie in een spiraal belanden van voortdurend afnemende investeringen en stagnerende groei. De grootte van de 'corridor' hangt mede af van het gevoerde begrotingsbeleid.

In de hoofdstukken 6 en 7 richt de analyse zich op de wisselwerking tussen de (financiële) ontwikkeling van de overheid, de bedrijvensector (inclusief de eigenaren van de bedrijven) en de werknemers van de bedrijven *op lange termijn*. In hoofdstuk 6 wordt aangetoond dat zich een 'financiële' cyclus kan voordoen, waarbij fasen van optimisme, waarin bedrijven sterk expanderen en hun schulden uitbreiden, zich afwisselen met fasen waarin bedrijven een meer behoudende financiële strategie volgen. Hoofdstuk 7 laat zien dat een open economie een nauwe samenhang bestaat tussen de schuldontwikkeling van de overheid en de vermogenspositie van het land ten opzichte van het buitenland.

De centrale vraag in de macro-economische hoofdstukken 6 - 8 betreft de stabiliteit van de economie. Daarom wordt een methode ontwikkeld om te bepalen in hoeverre verschillende factoren een stabiliserende, dan wel een de-stabiliserende invoed op de economie hebben. De theoretisch resultaten worden ondersteund met simulaties, die worden uitgevoerd op basis van een numerieke invulling van de modellen.

Op de middellange termijn blijkt de stabiliteit vooral af te hangen van de mate van prijsflexibiliteit, de invloed van monetaire variabelen op de bestedingen, en de mate waarin investeringen reageren op wisselingen in de bezettingsgraad van de produktiecapaciteit. Op de lange termijn hangt de stabiliteit af van een veelheid van factoren. Opvallend is dat in onze studie het positieve effect van het vermogen op de consumptie een nadelige invloed heeft op de stabiliteit van de economie, dit in tegenstelling tot veel andere studies waar een sterk vermogenseffect juist een essentiële voorwaarde is voor stabiliteit.

In de open economie hangt de stabiliteit met name af van de hoogte van de reële rente en het groeitempo van de economie. Aangezien de reële rente grotendeels door de internationale rente wordt bepaald, vormt de economische groei de belangrijkste binnenlandse determinant van de stabiliteit. Dit betekent, dat in het algemeen het risico van een instabiele ontwikkeling van de staatsschuld (en de buitenlandse schuld) aanmerkelijk groter voor een traag groeiende economie dan voor een economie met een hoog groeitempo.

Met betrekking tot het overheidsgedrag worden verschillende beleidsregimes onderzocht, zoals die in de literatuur bekend zijn. Deze regimes worden onderscheiden naar hun doelstelling, zoals een vaste norm voor het financieringstekort, een norm voor de staatsschuld, een bepaald niveau van overheidsbestedingen en belastingen, of een norm voor de som van bestedingen en rentelasten. De keuze van het regime blijkt grote consequenties te hebben voor de stabiliteit van de economie. Het is echter niet mogelijk om een eenduidige rangorde op te stellen van de regimes naar de mate waarin zij stabiliserend of de-stabiliserend zijn. De invloed van een beleidsregime op de economie hangt sterk af van de omstandigheden, en van de termijn die men in beschouwing neemt. Zo blijkt het regime dat uitgaat van een vaste norm voor de omvang van de staatsschuld, het meest stabiele regime te zijn in de lange termijn, terwijl het in de middellange termijn juist het meest de-stabiliserende regime is.

Hoewel het, zowel op politieke als op economische gronden, aantrekkelijk kan zijn om regels of normen te hanteren, toont deze studie aan dat regels en normen voor het voor het begrotingbeleid van de overheid altijd gebonden zijn aan de economische omgeving. Een verandering in de omstandigheden, geleidelijk of schoksgewijs, kan voorheen stabiele beleidsregimes veranderen in instabiele regimes. Beleidsregimes moeten derhalve regelmatig worden heroverwogen, en zonodig worden aangepast.